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Some Aspects of Perception

I: SIZES OF HARMONIC INTERVALS IN PERFORMAN

There is a certain marvellous order which belongs to the nature of harmony in general; in this order every instrument, to the best of its ability, participates under the direction of that faculty of sense-perception on which they, as well as everything else in music, finally depend.

Aristoxenus, The Harmonics

Thoughtful consideration of the tonal material of music, its theory, notation, and performance must always lead to the question of its auditory image, the way it is actually perceived by the human mind. Quantitatively, the auditory image can be expressed in the frequencies of tones in cycles per second and the sizes of intervals, melodic and harmonic, in cents. In Western music, theories of tonal material, from Euclid*1 on, have provided systems of interval relationships from which

By

CHARLES

SHACKFORD

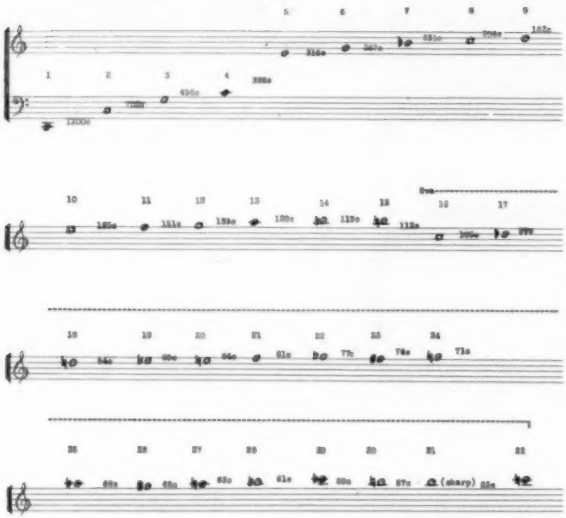
When you can measure what you are speaking about and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager, unsatisfactory kind.

Lord Kelvin

scales of tones of fixed pitch can be developed. This is especially valuable to the construction of musical instruments and helps to determine where to drill holes in the aulos or the oboe or how to tune the strings of the kithara or the clavichord. History shows, however, that advances in the art have a way of outmoding such systems. Thus John Bull's Fantasy on the Hexachord,*2 with its enharmonic modulation from F# to Gb, exceeded the meantone tuning of Pietro Aron*3 and this and

example 1

THE HARMONIC SERIES FROM 1 TO 32
INTERVAL SIZES IN CENTS



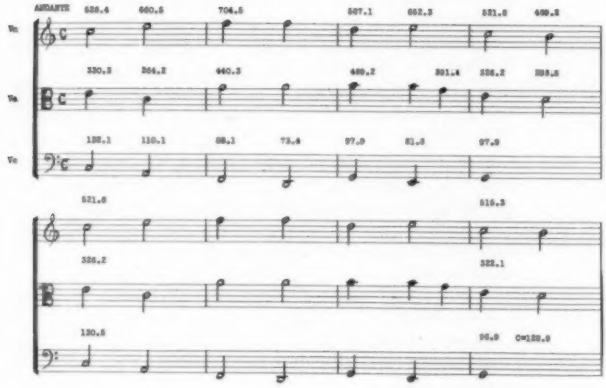
THE RESULTANT TONE EFFECT
AN AUDITORY PHENOMENON

similar speculative essays led to the adoption in keyboard tuning of the equal temperament which had been used for many decades in the tuning of fretted instruments.

For practical reasons tuning systems are forced to confine tonal material. Bosanquet's division of the octave into 53, while offering admirable freedom, has no practical application to the construction of musical instruments.*4 What quantitative information about the perception of tonal material we possess is almost entirely based on the proportions and interval sizes of theoretical systems, while its actual standard is the piano which, even expertly tuned, is apparently an imperfect example of equal temperament.*5 But no thoughtful musician is satisfied with the restrictions of the piano or the system of equal temperament as an explanation of the tonal material of music. Sooner or later in a discussion of the problems of enharmonic notation or the resolution of tritones, someone will observe that string players adjust pitch to circumstances. This being so, they must produce a kind of temperament in performance which is the most satisfactory realization of tonal material from the artistic point of view and, at the same time, a reliable reflection of its auditory image, even taking into account such partially fulfilled intentions as may result from human error. Numerical measurements of the pitches and interval sizes of expert string ensemble performance without open strings, can provide the quantitative information needed for anything more than a speculative discussion of human perception of tonal material and its relation to music theory.

But the problems of obtaining reliable numerical measurements of this type have always been considerable and, beginning with Delezenne*6 in the 1820's, the rewards have seemed sufficiently attractive to only a handful of researchers, none professionally engaged in music. The data presented in this article resulted from a research project initiated when the writer was a candidate for the doctorate in music at Harvard University and continued on a postdoctoral fellowship. Essential to this kind of undertaking are a reliable research method of sufficient accuracy and subjects of sufficient experience and ability to know what they want and how to produce it. To satisfy the first condition a method was established by which the pitch of each note of each instrument in a string trio could be obtained from a single performance to an accuracy of somewhat better than 1/10 of a cycle. The participants, members of three string trios, were all expert performers who had had considerable experience playing in their respec-

example 2

HYPOTHETICAL PERFORMANCE IN JUST INTONATION

tive groups. Those in the trios designated A and C were members of the Boston Symphony Orchestra, who also played chamber music professionally, while the professional work of the members of B was almost entirely in chamber music.

The details of the method of obtaining the data are outside the scope of this article and will be reported elsewhere. The important aspect, as stated above, is that separate records in cycles per second of the pitches produced by each of the three players could be obtained in a single performance of each piece. From these, the sizes in cents of the actual harmonic and melodic intervals of a particular performance were computed according to the method of Ellis.*7 Determining the pitches, which involved actual handcounting of wave peaks within a time interval on 35mm. film, as well as calculations to determine fractions of cycles, was a time-consuming process. However, due to frequency modulation caused by the use of vibrato on almost every note, automatic counting devices could not be used. To eliminate frequency modulation as a source of error, a study was made of the perceived pitch of the vibrato tone and the results reported in another periodical.*8

Any discussion of the tonal material of music must take into consideration certain basic aspects of its physical properties. There is the overtone series, as shown in Example 1, the natural harmonic division of a tone which yields all the common intervals of music and is a demonstrable auditory phenomenon. The purpose in carrying it as far as 32, which is slightly more than an equal tempered quarter tone above 31, will become apparent later. The distances between the tones are shown in cents to convey the feeling of the diminishing distance between adjacent tones as the series ascends. An interesting feature of the harmonic series is that the difference in pitch between adjacent tones is always the pitch of the fundamental tone while the difference between any two pitches in the series will be another pitch in the series. This leads to a consideration of the resultant tone effect, another demonstrable auditory phenomenon which provides a remarkable means of testing the acoustical purity of musical intervals. By resultant tone is meant the tone whose pitch is the difference between those of the two sounding tones by which it is produced in the human ear. From the auditory point of view it is by far the most important of the combination tones produced by a harmonic interval. It is most clearly heard when the sounding tones are free of strong harmonics and less than an octave apart. Under these circumstances, the resultant tone is below the sounding

tones and can be very intense, as flute players know. The effect is a commonplace in organ building. As a means of testing the size of a harmonic interval, it is most effective when it is the duplicate of one of the sounding tones and as close to that tone as possible. The fifth best fulfills these conditions, the reason this interval was adopted for open string tuning. If the fifth is not exactly the size it is in the overtone series, the resultant tone will not form a true octave with the lower tone of the sounding interval. Other intervals are less well defined in this respect and there is a hierarchy which is an important element of Paul Hindemith's evaluation of musical raw material.*9

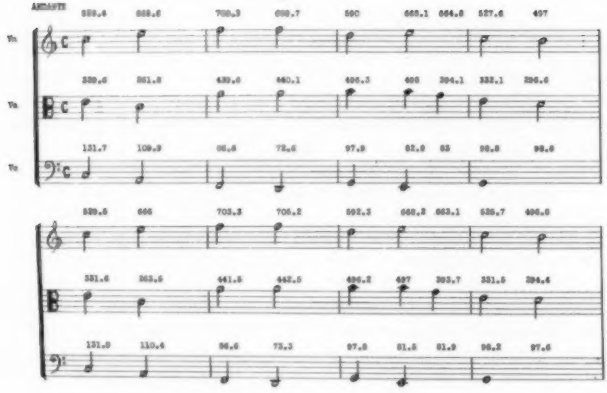
Because resultant tones produce other resultant tones with the sounding tones and with each other, the number theoretically present in a complex vertical construction being almost infinite, some writers refuse to consider the phenomenon as important to music theory. In this connection it is important to bear in mind that the first resultant tone produced by an interval is very much stronger than the next, while the following ones are successively weaker. The author found this to be the case with resultant tones produced by sounds from audio oscillators in the physics laboratory at Harvard. Tones from these instruments are relatively free of harmonics and, when two of different pitch are combined, produce clear resultant tones. The case of the tritones shown in Example 1 was typical of other intervals. The second order resultant tones (shown as black notes) came from the combination of the resultant tone with each of the sounding tones and were barely audible, while the resultant tone itself was strong and clear, demanding attention and making a three part harmony where only two tones were actually sounding. The importance of the first order resultant tone as an auditory phenomenon has been noted by Wever*10 and actual measurements of amplitudes in the human ear have been made by Békésy*11. The latter study shows that resultant tones in the range 200-800 cycles per second are about 40 db weaker than the sounding tones; resultant intensities increase as sounding tone intensities increase; resultant tones below 200 cps become more intense relative to the sounding tones and at 100 cps are four times as strong as they are in the region 200-800 cps. As the vast majority of resultant tones in musical performance lie below 200 cps this last point is of prime importance.

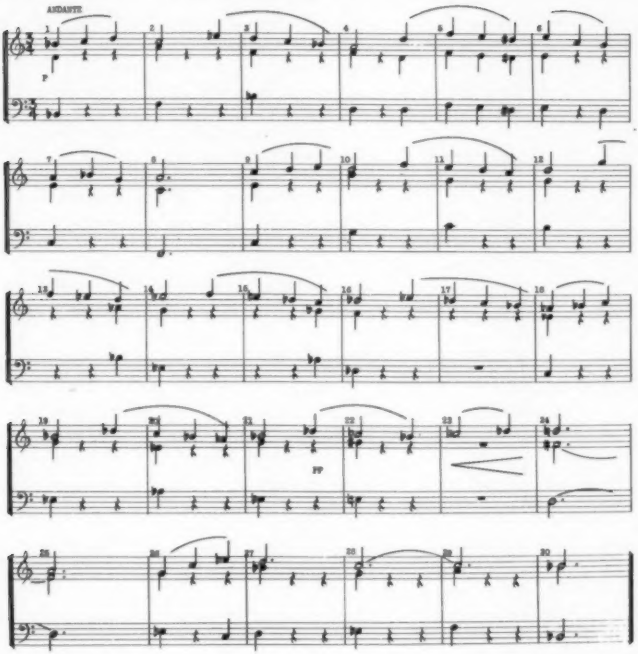
The tidy picture presented by the harmonic series and the resultant tone effect offers the almost irresistible temptation to suppose that herein lies the true basis of tonal material and

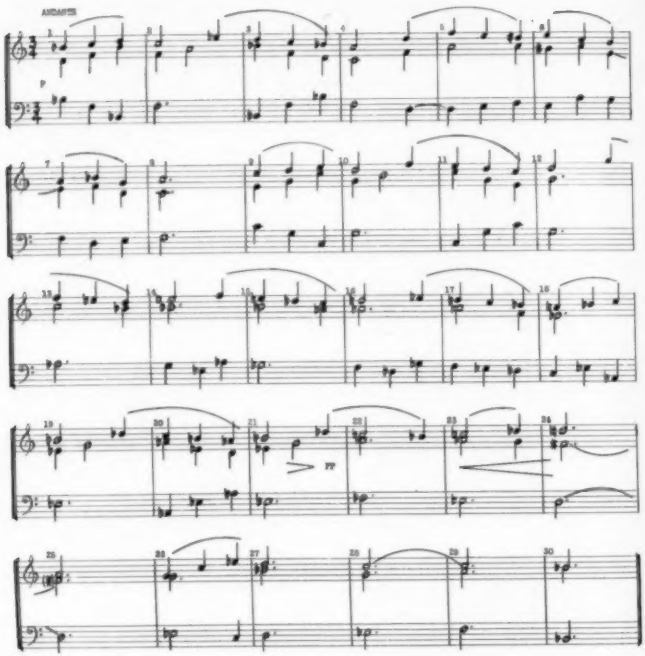
that anything else is a falling away from a perfection provided by nature. But the extent to which tonal perception is based on the physical facts of tonal material and the extent to which it is based on the way tonal material is used in music is a question that must always concern the music theorist. It is a fact that troubles appear as soon as the intervals of the harmonic series are used in music. There is already a flaw in the just intonation major scale derived from the series; its second step cannot be reconciled with both its fifth and sixth steps at the same time. This anomaly has called forth demonstrations such as shown in Example 2 where the pitch drops the syntonic comma, 80/81, in each performance even though all the intervals are exactly those in the series. An actual performance of this twice-played phrase (see Example 3) shows irregularities but no actual lowering of pitch. The numbers over the tones indicate their frequencies. While just intonation is not, of course, the only system that presents problems of this kind, it is a question whether string players, who are for the most part free to produce the intonation they want, are always satisfied with the strictures of equal temperament when the music suggests something else.

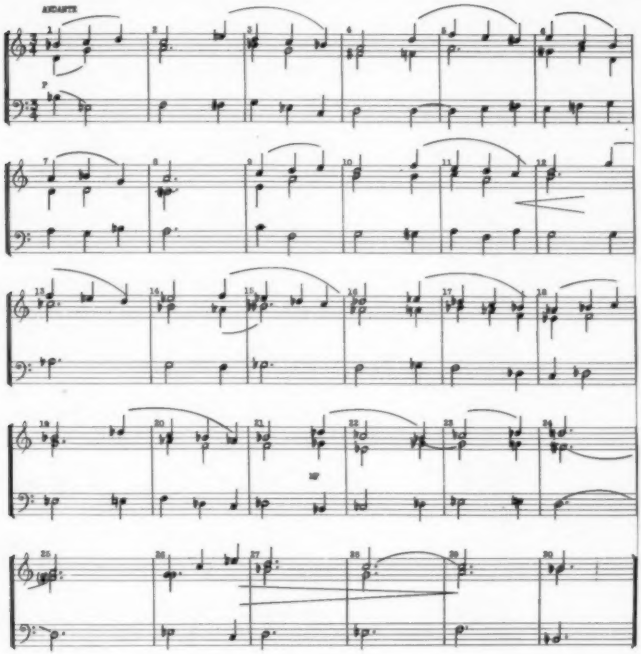
The three trios participating in the study whose results are about to be considered, were asked to perform four short pieces composed by the author. The pieces (see Examples 4 to 7) are different settings of the same melody, each embodying features to be studied. Setting I allows the violinist some freedom while he is always responsible to other parts in the remaining settings. The horn-like progressions in Setting II provide the best possible circumstances for the production of major thirds and fifths and for the employment of 9/10 whole steps between scale steps 2 and 3 and 8/9 whole steps between scale steps 1 and 2; a 4/5 third and 2/3 fifth between the viola and violin in m. 1 or 3 would require these melodic steps in the violin part. This setting also has enharmonically equivalent tritones in the dominant seventh and augmented sixth chords in m. 2 and 5, and 19 and 23 respectively. In m. 23, the tritone is spelled as a diminished fifth though used as an augmented fourth, the purpose of this deception being to illuminate somewhat the question of the relationship of the sound of music to its notation. If there were a difference between the tritones in m. 2 and 5, then one might expect that, once the progression in m. 23 and 24 had been discovered for what it really was, the tritone in m. 23 would not be played the same as it was in m. 19. Believers in just intonation would maintain that correctly played, the violin tied C would be an 80/81 comma higher in m. 29 than in m. 28 in order to be in

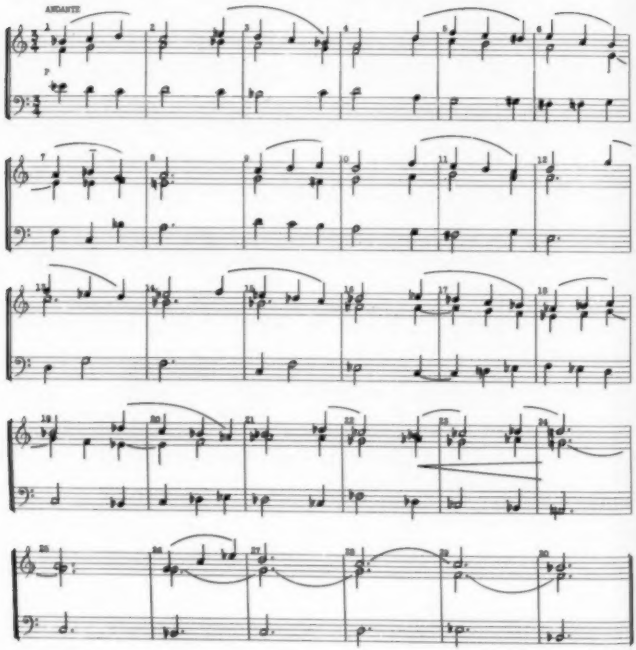
example 3

ACTUAL PERFORMANCE

example 4

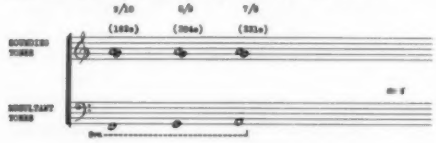
SETTING I (VIOLIN, VIOLA, AND CELLO)

example 5

SETTING II (VIOLIN, VIOLA, AND CELLO)

example 6

SETTING III (VIOLIN, VIOLA, AND CELLO)

example 7

SETTING IV (VIOLIN, VIOLA, AND CELLO)

tune first with the II chord and then with the V chord. The long-held tones give players the opportunity to do this if they feel it is required. Setting III provides a number of enharmonically equivalent tritones including, in m. 13 and 15, augmented fourths between the upper parts which are not resolved as such. Measures 19 and 23 duplicate the circumstances in Setting II described above. The independent use of dissonant intervals is the main feature of Setting IV. Chords of two fourths, passing and long-held on important points, are used. A chord D^{b}-A^{b}-B^{b} from the bass appears at a half cadence in m. 21 and as part of a progression in m. 22. A long-held second, G-A in m. 25, gives ample time for the players to decide about the distance between the two tones. As the resultant tone effect has time to register and the one produced by G and A would be the lowest perceived sound, the size of the harmonic second could be an important matter. An 8/9 interval would produce a G reinforcing the feeling of a C chord, while 9/10 would produce F which with the other tones, sounding and resultant, would convey the feeling of an F chord. If the interval were as large as 7/8, the lowest resultant tone would be A, the actual root of conventional analysis (see Example 8). The four pieces present no technical problems and, being in a register where everything can be clearly heard, provide favorable circumstances for the solution of their intonational problems. Careful rehearsals preceded recording sessions. No open strings were used.

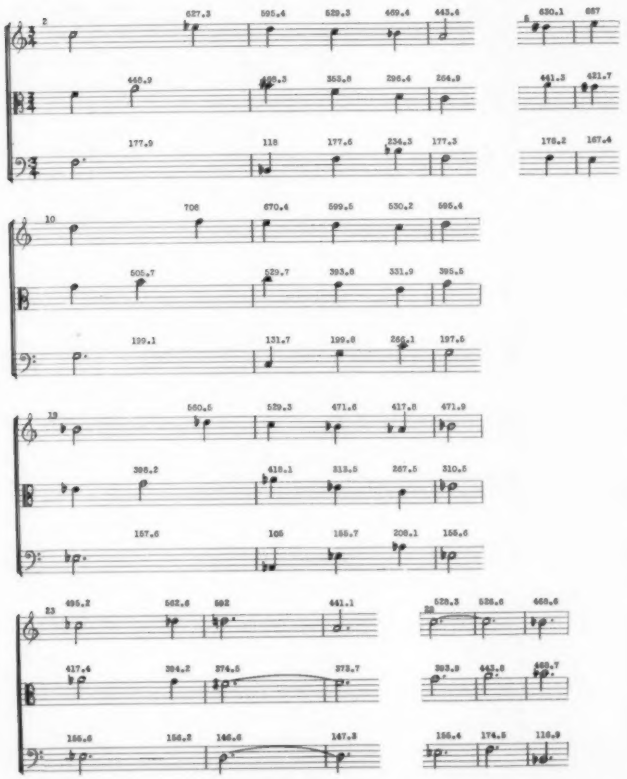
example 8

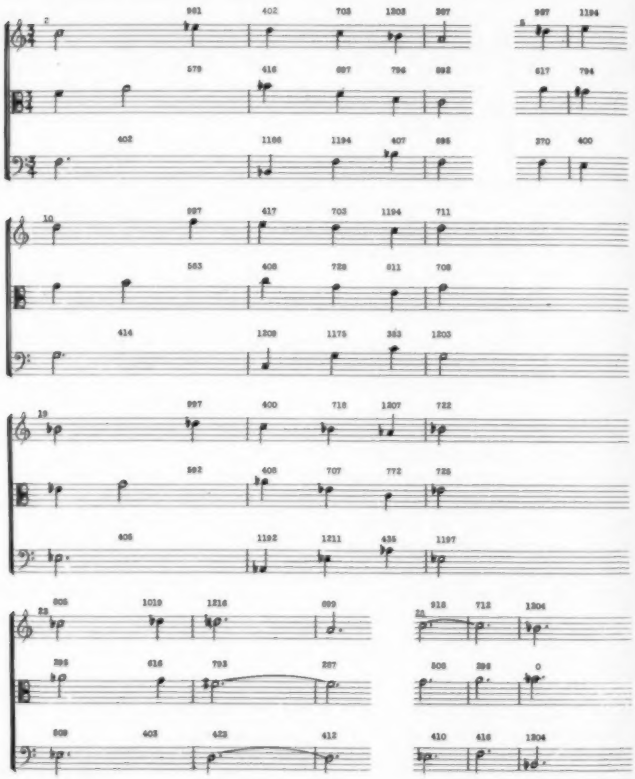
RESULTANT TONES PRODUCED BY DIFFERENT SIZES OF HARMONIC WHOLE STEPS

Since no open strings were used, it is important to consider how well the groups managed to maintain the tonic pitch from beginning to end with only their ears to guide them. In the four performances of Setting II in which both the violin B^b in m. 3 and that in m. 30 were analyzed, the differences between their pitches were as follows: A2 - 0 cents, B1 - 6 cents, B2 - 3 cents, C3 - 2 cents. In spite of passages in two other keys, C and A^b, and an enharmonic modulation, the feeling for the tonic was sufficiently strong to show a tolerance between the two B^b's as small as that of the long-held unison.

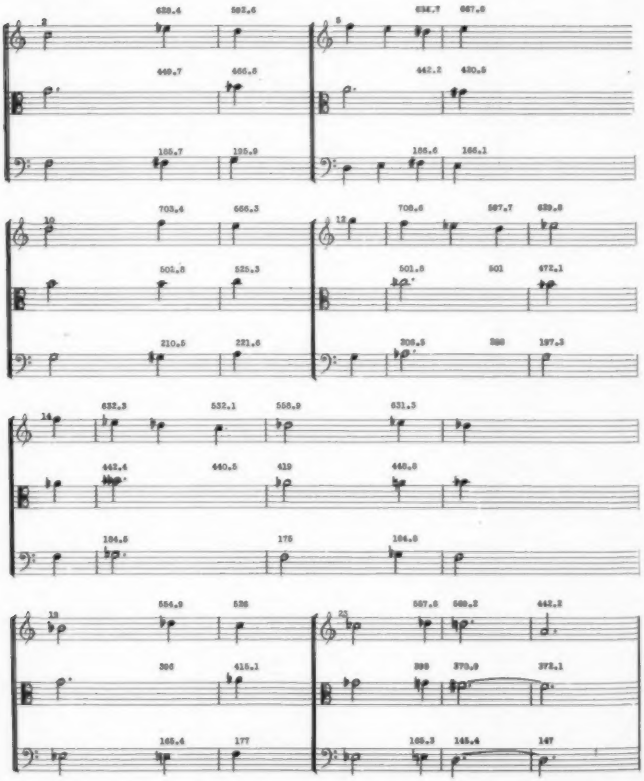
As an aid to the reader, frequencies of tones and sizes of harmonic intervals in cents for sections of performances of Settings II, III and IV are shown together with the music in Examples 9 to 16. In the tables, which in each case present the data of all the performances, Roman numerals refer to the setting, capital letters followed by numbers to the trio and the number of the performance.

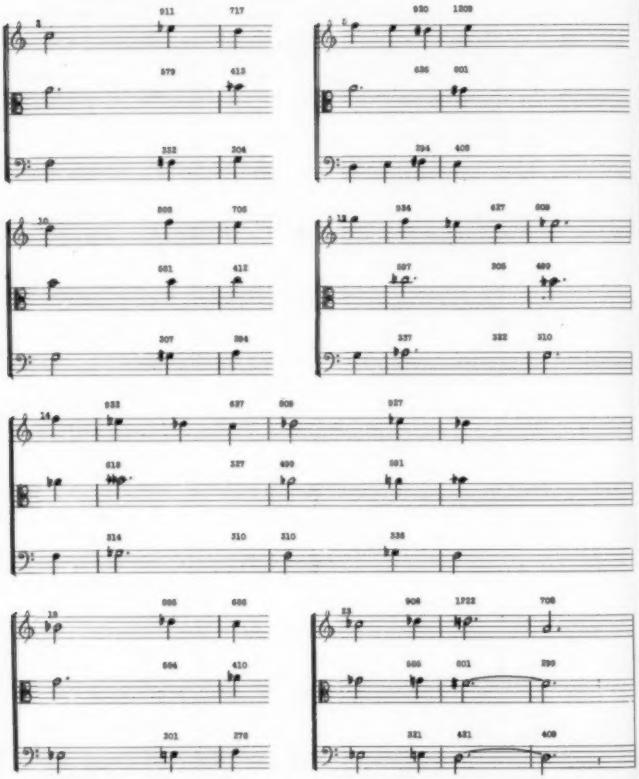
In considering the numerical data which follows, it is important to have a feeling for the limits of discrimination of the players so that what was apparent and intentional, and thus significant information about tonal perception, may be separated from differences that were too fine to be the result of anything but chance. A good starting point is the unison in the last measure of Setting II. The distances between the viola and violin notes in the four performances analyzed were as follows: A2 - 4 cents, B1 - 2 cents, B2 - 0 cents, and C3 - 3 cents. The tolerance of this interval is apparently quite small, as one might expect. Determining the tolerances of other intervals presents a problem. Tolerance may vary not only with different intervals but also in the way they are used in music. One suspects the tolerance of long-held harmonic intervals, which really have a chance to be heard, to be smaller than that of intervals used in passing. For most cases it is probably safe and conservative to regard 10 cents, or about half the syntonic comma, 80/81, as the threshold of significance; differences as great or larger are considered as calculated, while smaller differences are progressively less so.

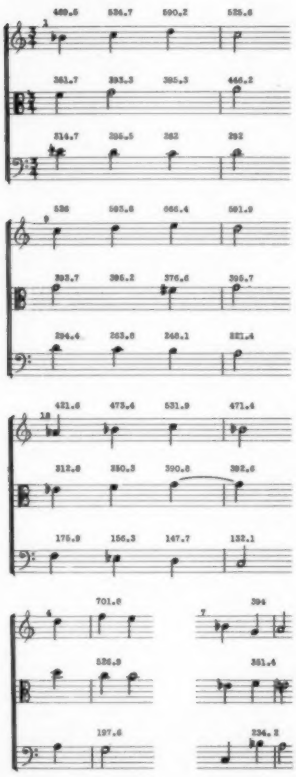
example 9

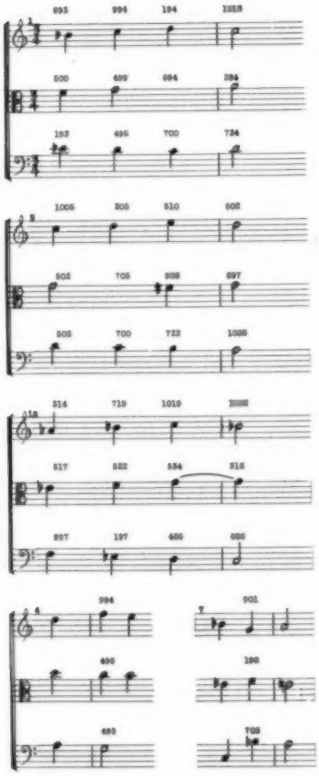
SETTING II, PERFORMANCE B2
FREQUENCIES OF TONES IN CYCLES PER SECOND

example 10

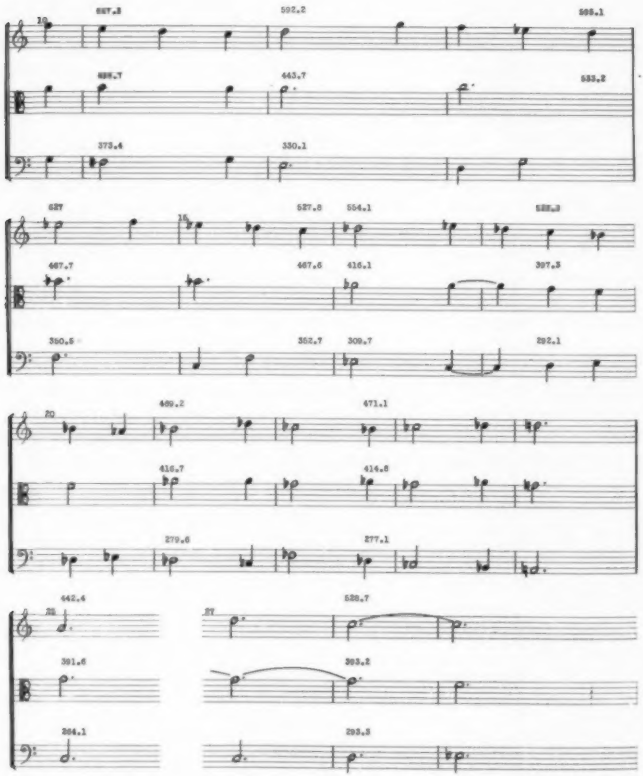
SETTING II, PERFORMANCE B2
SIZES OF HARMONIC INTERVALS IN CENTS
(NUMBERS ABOVE VIOLIN PART INDICATE DISTANCE BETWEEN OUTER VOICES)

example 11

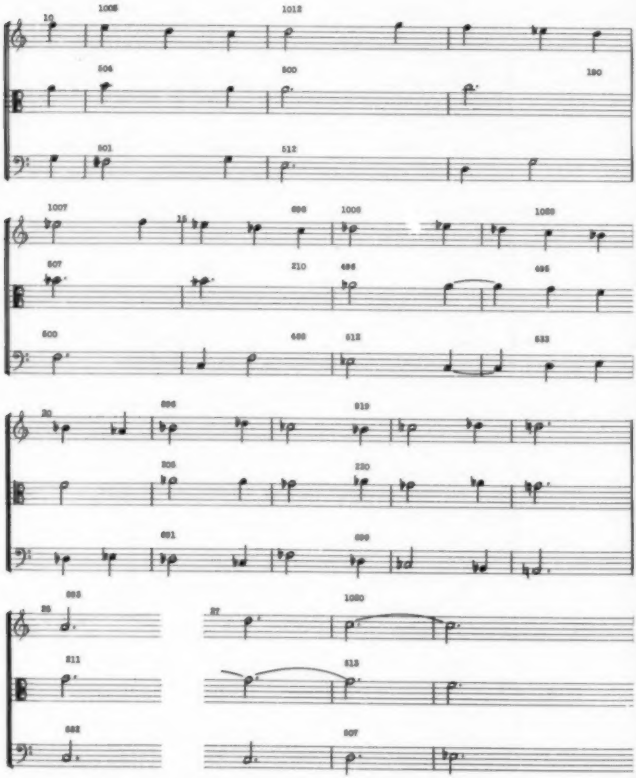
SETTING III, PERFORMANCE B2
FREQUENCIES OF TONES IN CYCLES PER SECOND

example 12

SETTING III, PERFORMANCE B2
SIZES OF HARMONIC INTERVALS IN CENTS
(NUMBERS ABOVE VIOLIN PART INDICATE DISTANCE BETWEEN OUTER VOICES)

example 13

SETTING IV, PERFORMANCE B2
FREQUENCIES OF TONES IN CYCLES PER SECOND

example 14

SETTING IV, PERFORMANCE B2
SIZES OF HARMONIC INTERVALS IN CENTS
(NUMBERS ABOVE VIOLIN PART INDICATE DISTANCE BETWEEN OUTER VOICES)

example 15

SETTING IV, PERFORMANCE B2
FREQUENCIES OF TONES IN CYCLES PER SECOND

example 16

SETTING IV, PERFORMANCE B2
SIZES OF HARMONIC INTERVALS IN CENTS
(NUMBERS ABOVE VIOLIN PART INDICATE DISTANCE BETWEEN OUTER VOICES)

Fifths and twelfths are shown in Examples 17 and 18; and then a statistical analysis of the sizes of these intervals in performance is represented in Example 19. The spread of 50 cents, a quarter tone, between the largest and smallest fifths played is surprisingly large for the interval that is commonly supposed to be the most sensitive to inaccuracies of intonation. However, the quartiles show that when all the data was arranged in numerical order, the central 50 per cent had a spread of only 15 cents. The median, 705 cents, which is the exact middle of the numerical arrangement of the data is in very close agreement with the arithmetic mean or average size of 707 cents. Looking more closely, it can be seen that fifths in Setting II, which is in a simple, easily grasped harmonic style, had a very much smaller spread than those in Setting IV in which seconds and sevenths play an independent and important role in the harmonic life of the music. This tendency is apparent in the quartile values as well; a 15 cent spread in the case of Setting IV and one of only 9 cents in that of Setting II. The wider latitude of fifths in a dissonant style can also be seen in the analysis of passing and long-held intervals. The smallest spread for a group is that of the 11 long-held fifths in Setting II. This may very well be because the situations in which these intervals occur are relatively simple and unconfused, and the physical, acoustical properties of the tonal material have the opportunity to assert themselves. It can be seen, for example, that the fifths most nearly approaching the natural value of 702 cents are those at the full cadence in m. 8 of Setting II. The twelfths show the same tendency as the fifths. The spread of those in Settings II and III is less than that in Setting IV, but at the same time almost significantly greater than that between all fifths in Setting II. This may be due to the greater than an octave gap between the two tones, which would cause the resultant tone to fall between them and be perceived not so easily as in the case of the fifth.

example 17

SIZES OF HARMONIC FIFTHS IN CENTS

Measures and Notes	Setting, Trio and Performance				
	II				
	A2	B1	B2	C3	
3 Va. F - Vn. C	711	703	697	702	
4 Vc. F - Va. C	705	693	695	717	
8 Vc. F - Va. C	704	-	697	704	
11 Va. G - Vn. D	705	692	728	704	
12 Va. G - Vn. D	715	703	708	721	
20 Va. E^b - Vn. B^b	693	709	707	720	
21 Va. E^b - Vn. B^b	715	708	725	711	
	IV				
	A2	A3	B2	C1	C2
1 Vc. E^b - Vn. B^b	716	712	693	-	704
Va. G - Vn. D	728	-	694	-	699
Vc. C - Va. G	684	-	700	-	714
2 Vc. D - Va. A	695	-	734	-	723
7 Vc. B^b - Va. F	-	718	703	712	689
9 Va. G - Vn. D	-	-	705	-	714
Vc. C - Va. G	-	-	700	-	700
Vc. B - Va. F#	-	-	722	-	709
10 Va. G - Vn. D	709	-	697	-	704

example 18

SIZES OF HARMONIC TWELFTHS IN CENTS

Measure and Note	Setting, Trio and Performance			
	II			
	A2	B1	B2	C3
25 Vc. D - Vn. A	710	685c	699c	719c
29 Vc. F - Vn. C	711	695	712	720
	III			
	A2	B2	C2	
3 Vc. G - Vn. D	710	717	704	
11 Vc. A - Vn. E	-	706	723	
20 Vc. F - Vn. C	716	686	707	
25 Vc. D - Vn. A	726	708	723	
	IV			
	A1	A2	A3	B2	C1	C2
13 Vc. G - Vn. D	-	705	-	-	-	717
15 Vc. F - Vn. C	-	712	-	698	-	730
18 Vc. E^b - Vn. B^b	-	718	-	719	-	733
19 Vc. C - Va. G	-	-	-	686	-	719
21 Vc. D^b - Va. A^b	715	702	708	691	714	685
22 Vc. D^b - Va. A^b	684	682	701	699	722	702
25 Vc. C - Va. G	722	733	714	682	708	680

example 19

STATISTICAL ANALYSIS OF HARMONIC FIFTHS AND TWELFTHS

	Number of Data	Spread	Extremes	Quartiles	Median	Mean
All fifths	53	50c.	684-734c.	699-714c.	705c.	707c.
Fifths in II	27	33	692-725	702-711	705	706
Fifths in IV	26	50	684-734	699-714	704	707
Passing fifths in II	16	36	692-728	702-713	707	708
Passing fifths in IV	20	44	684-728	699-714	704	706
Long-held fifths in II	11	28	693-721	-	704	706
Long-held fifths in IV	6	39	695-734	-	707	710
All twelfths	47	53	680-733	699-719	710	708
Twelfths in II & III	19	41	685-726	704-719	710	709
Twelfths in IV	28	53	680-733	694-718	708	706

Fifths: Pythagorean and natural, 702 cents;
meantone, 696 cents; equal tempered, 700 cents.

Thirds and tenths are shown in Example 20 and are then analyzed in Example 21. Major thirds and tenths have about the same spread but the median and mean is larger for thirds. Since most of the major thirds occurred between the viola and violin and most of the tenths between the cello and the viola, these intervals were larger when the violin was a participant than when the two lower parts were involved. Scrutiny of Example 20 indicates that this effect was common to all three trios. As with fifths, long-held thirds and tenths show a narrower spread than those used in passing. Though sizes approaching the natural value of 386 cents were used, the Pythagorean interval of 408 cents appears to be the most representative of actual practice. It is interesting to observe that long-held major thirds and tenths are the smallest in size. As with the fifth, the longer the interval is held the more apparent its acoustical properties become. The tenth D - $F^{\#}$ in m. 25 of Settings II and III is in five cases smaller than it was in m. 24. In II A2 a value of 397 cents, that of equal temperament, was apparently satisfactory; but III C2 achieved the acoustically nearly perfect interval of 389 cents and one wonders if perhaps the tendency to reduce the size of a large major third with time originates with the resultant tone effect and that, long enough held, these intervals in the hands of sensitive musicians would all be nearer to 386 cents than to 400 cents. To be sure, in III A2 the size of the interval increased with time, but this is only one of seven cases.

The few minor thirds show two quite different attitudes toward this interval. Trio B, the chamber musicians, preferred a value which was most of the time substantially below the equal tempered interval of 300 cents, while Trios A and C, who were doing a large amount of orchestral work, preferred a value substantially higher. These two facts may, of course, not be related and the amount of data is quite small. Nevertheless, m. 23 in Setting II provides a good opportunity to produce the minor third that is wanted. The commonly accepted natural size of the minor third is the 5/6 interval in the overtone series. The resultant tone of this interval after octave transposition is a fifth below its upper tone which is fine when the interval is part of a major triad, but produces a

confusing situation in minor triads. This has lead Hindemith to believe that nature has made the minor triad less clear than the major.*9 But actually, nature has been a little kinder by providing a minor third, between 16 and 19 in the harmonic series, whose resultant tone confirms the lower tone as a root and the minor character of the interval. The two minor thirds are presented in Example 22. It is interesting to note that the major third forming a perfect fifth with the minor third 16/19 is 404 cents. The resultant tone effect suggests the larger minor third in major triads and the smaller one in minor triads, but the small amount of data presented here indicates no such tendency. In view of the preference for large major thirds, the equal tempered minor third is actually a very good solution.

The whole steps presented in Example 23 and then analyzed in Example 24 are used as independent intervals in Setting IV. The spread of 56 cents for all whole steps is wide considering the size of the interval. A majority resolve by stepwise contrary motion into fourths and these have a larger median and mean size than the few whole steps which occur in more or less static circumstances. The width of the range of the latter group may result from two different interpretations of the whole step in the chord of m. 25. The root of this chord is a question that was raised by Rameau*12 and the position of the whole step as a possible determinant was discussed above in connection with Example 8. The data shown in Example 23 indicate that Trio A preferred the interval small while Trios B and C tended to a large interval. A3 produced a resultant tone G which would strengthen the feeling of C as the chord root. The whole step in A2 is almost exactly 9/10 and would have produced F as a resultant tone, while A1 may have been leaning in this direction but is certainly less definite than either A2 or A3. B2 and C1 would have produced a slightly flat C as resultant and may have chosen to group their tones around C. But C2 very definitely produced the 7/8 whole step whose resultant tone would have been a good A and a very reasonable note, in terms of conventional analysis, around which to group the others. This is not as farfetched as it may appear to some. The chord lasted long enough to be very well heard and definite decisions about the whole step appear to have been made in at least three cases.

Measures 21 and 22 offer a comparison between static and resolving whole steps. In three cases the resolving whole step in m. 22 is significantly larger than the static whole step at the half cadence in m. 21. In C1 the 8 cent difference

example 20

SIZES OF HARMONIC MAJOR AND MINOR THIRDS AND TENTHS IN CENTS

Measure and Note	Setting, Trio and Performance						
	II				III		
	A2	B1	B2	C3	A2	B2	C2
Major Thirds							
3 Va. B^b - Vn. D	401	418	416	419	416	413	413
Vc. B^b - Va. D	-	397	407	387	-	-	-
11 Va. C - Vn. E	395	405	408	414	402	412	423
Vc. C - Va. E	393	396	383	410	-	-	-
20 Va. A^b - Vn. C	406	422	408	420	407	410	418
Vc. A^b - Va. C	-	444	435	417	-	-	-
Major Tenths							
4 Vc. F - Vn. A	411	397	387	423	-	-	-
5 Vc. F - Va. A	424	407	370	404	-	-	-
6 Vc. E - Va. G#	406	429	400	417	393	408	416
8 Vc. F - Vn. A	404	-	390	415	-	-	-
24 Vc. D - Va. F#	397	407	423	411	396	421	400
25 Vc. D - Va. F#	397	399	412	403	417	409	389
28 Vc. E^b - Va. G	407	398	410	398	-	-	-
29 Vc. F - Va. A	396	420	416	402	-	-	-
Minor Thirds							
23 Va. A^b - Vn. C^b	326	288	296	325	-	-	-
25 Va. F# - Vn. A	313	286	287	316	309	299	334
29 Va. A - Vn. C	315	275	287	318	-	-	-
Minor Tenths							
3 Vc. G - Va. B^b	-	-	-	-	294	304	291
11 Vc. A - Va. C_b	-	-	-	-	-	294	300
20 Vc. F - Va. A^b	-	-	-	-	309	276	289

STATISTICAL ANALYSIS OF HARMONIC THIRDS AND TENTHS

	Number of Data	Spread	Extremes	Quartiles	Median	Mean
All major thirds	31	61c.	383-444c.	402-418c.	410c.	410c.
All major tenths	40	59	370-429	397-415	406	406
Thirds & tenths Vc.-Va.	43	74	370-444	397-416	407	406
Thirds & tenths Vc., Va.-Vn.	28	36	387-423	404-416	411	410
Passing thirds & tenths	42	74	370-444	402-417	409	409
Long-held thirds & tenths	29	36	387-423	397-413	404	405
Minor thirds	15	59	275-334	287-318	309	305
Minor tenths	8	33	276-309	-	294	295

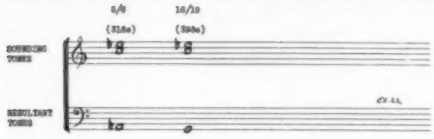
Major third: Pythagorean 408 cents; natural and meantone 386 cents; equal tempered 400 cents.

Minor third: Pythagorean 294 cents; natural 5/6, 316 cents; natural 16/19, 298 cents; meantone, 310 cents; equal tempered, 300 cents.

examples

22

MINOR THIRDS IN THE HARMONIC SERIES



23

SIZES OF HARMONIC WHOLE STEPS AND NINTHS IN CENTS

Measure and Note	Setting, Trio and Performance IV					
Whole Steps	A1	A2	A3	B2	C1	C2
1 Vc. E^b - Va. F	201	188	210	193	214	213
7 Va. F - Vn. G	181	203	197	198	214	207
13 Va. C - Vn. D	208	205	210	190	212	197
15 Va. B^b - Vn. C	206	210	201	210	217	209
21 Va. A^b - Vn. B^b	200	196	188	205	200	190
22 Va. A^b - Vn. B^b	196	191	208	220	208	213
25 Va. G - Vn. A	193	179	198	211	212	235
Ninths						
Vc. C - Vn. D	-	212	-	194	-	213
9 Vc. C - Vn. D	-	204	-	205	-	214
18 Vc. E^b - Va. F	-	206	-	197	-	207

natural 16/15, 298 cents; meantone, [illegible]
tempered, 300 cents.

example 24

STATISTICAL ANALYSIS OF HARMONIC WHOLE STEPS

	Number of Data	Spread	Extremes	Quartiles	Median	Mean
All whole steps	51	56c.	179-235c.	197-211c.	205c.	204c.
Whole steps resolving to fourth	39	39	181-220	197-212	207	205
Static whole steps	12	56	179-235	-	199	201

Whole step: Pythagorean, 204 cents; natural 7/8, 231 cents; natural 8/9, 204 cents; natural 9/10, 182 cents; meantone, 193 cents; equal tempered, 200 cents.

example 25

SIZES OF HARMONIC FOURTHS AND ELEVENTHS IN CENTS

Measure and Note	Setting, Trio and Performance						
	II				III		
	A2	B1	B2	C3	A2	B2	C2
28 Va.G - Vn.C	489	503	508	522	-	-	-
14 Va.B^{b} - Vn.E^{b}	-	-	-	-	493	499	497
16 Va.A^{b} - Vn.D^{b}	-	-	-	-	501	499	517

	IV					
	A1	A2	A3	B2	C1	C2
1 Va.F - Vn.B^{b}	515	528	502	500	485	491
Vc.D - Va.G - Vn.C	499-507	479-532	509-502	495-499	480-515	518-484
* 5 Vc.G - Va.C - Vn.F	499-492	492-515	-	498-496	490-491	510-481
9 Vc.D - Va.G - Vn.C	511-490	519-482	509-486	503-502	508-500	498-519
*10 Vc.A - Vn.D	-	507	-	502	-	491
*11 Vc.F#- Va.B - Vn.E	504-495	485-498	-	501-504	516-501	519-510
*12 Vc.E - Va.A - Vn.D	493-504	500-511	515-507	512-500	505-495	504-511
*13 Vc.G - Va.C	506	500	513	-	512	520
*14 Vc.F - Va.B^{b}- Vn.E^{b}	508-496	505-499	508-502	500-507	492-522	514-511
*15 Vc.F - Va.B^{b}	-	502	496	488	514	521
*16 Vc.E^{b}- Va.A^{b}- Vn.D^{b}	520-481	502-496	518-488	512-496	504-503	521-506
*17 Vc.D - Va.G - Vn.C	515-494	509-491	502-495	533-495	522-507	501-512
18 Va.E^{b}- Vn.A^{b}	-	497	-	517	-	498
Va.F - Vn.B^{b}	-	512	-	522	-	526
* Vc.D - Va.G - Vn.C	530-503	513-496	508-496	485-534	507-494	515-514
*28 Vc.D - Va.G - Vn.C	500-509	501-491	485-518	507-513	503-504	500-521

* Vc. and Va. or Vn. an eleventh apart

approaches significance in terms of the definition made earlier and indicates a tendency in full force in C2. It is more likely that melodic voice leading rather than acoustical forces exerted by the combination of tones produced the larger whole steps in m. 22 where the chord involved was passing and of short duration.

Fourths in Settings II, III, and IV are compared in Example 26. Although the data are few here it is worth noting that, as in the case of fifths, the simpler circumstances of II and III produced a smaller spread than the more complex harmonies of IV. The fourth also appears to have been generally larger in Setting IV. The intervals represented in Example 26 are single fourths used in combination with some other interval. Setting IV is very rich in chords of two fourths, which can be seen in Example 25. These are then analyzed in Example 27. Although in theory this is an ambiguous combination of sounds, yet in practice it takes on meaning for the composer and performer even if at a subconscious level. An examination of the data in Example 25 thus reveals that in a great number of these chords, one fourth is significantly larger than the other, some of the differences being really quite large. As Setting IV is clearly tonal music, an attempt has been made in Example 27 to relate the various sizes of fourths to some tonal function, in this case the relationship of the top note of the chord to the existing key center. The numbers indicate the amount in cents by which the fourth they represent exceeds the other fourth in the chord. When the upper note was a tonic it is fairly clear that the performers preferred the lower fourth to be larger. The differences vary and a few are not significant, but the trend is definitely there. The same is true with the upper note a mediant. The situation is not quite so clear when the upper note was a supertonic, but there is enough significant data to indicate that in this case the players wanted the upper fourth to be larger. There may be other, more subtle reasons for size differentiation in fourth chords and, indeed, the whole subject of interval sizes in complex chords deserves further investigation.

This study of the sizes of harmonic intervals in performance closes with a consideration of the tritone, the devil in music, regarded as the most ambiguous interval. Tritones in equal temperament are all the same size regardless of notation, and played on the piano they all sound the same and have no meaning until resolved. The harmonic series, on the other hand, contains two unequal tritones, 5/7 and 7/10, whose resultant tones, seen in Example 1 above, give direction and meaning

example 26

STATISTICAL ANALYSIS OF FOURTHS

	Number of Data	Spread	Extremes	Quartiles	Median	Mean
Fourths in II and III	10	33c.	489-522c.	-	500c.	503c.
Fourths in IV	12	43	485-528	-	507	508

Fourth: Pythagorean and natural, 498 cents; meantone, 503 cents; equal tempered, 500 cents.

example 27

DIFFERENCES IN SIZES OF INTERVALS IN FOURTH CHORDS

	Violin Note a Tonic					Violin Note a Supertonic				
A1	9	Vc.D	21	Va.G	Vn.C	Vc.D		Va.G	8	Vn.C
A2			37						53	
A3			25				7			
B2			1						4	
C1			8						35	
C2				21			34			
A1	14	Vc.F	12	Va.B^b	Vn.E^b	12 Vc.E		Va.A	11	Vn.D
A2			6						11	
A3			6				8			
B2				7			12			
C1				30			10			
C2			3						7	
A1	16	Vc.E^b	39	Va.A^b	Vn.D^b	28 Vc.D		Va.G	9	Vn.C
A2			6				10			
A3			30						33	
B2			16						6	
C1			1						1	
C2			15						21	

	Violin Note a Mediant									
A1	17	Vc.D	21	Va.G	Vn.C	18 Vc.D	27	Va.G		Vn.C
A2			18				17			
A3			7				12			
B2			38						49	
C1			15				13			
C2				11			1			

B2	38										
C1	15							13			
C2		11						1			

example 28

SIZES OF HARMONIC TRITONES IN CENTS

Measure and Note	Setting, Trio and Performance										
				I					II		
Diminished Fifth	A1	A2	A3	B1	B2	C1	C2	C3	A2	B2	C2
2 Va. A - Vn. E^b	598	595	608	567	579	600	593	593	603	579	603
10 Va. B - Vn. F	590	583	584	569	583	586	582	583	587	581	597
16 Va. A - Vn. E^b	-	-	-	-	-	-	-	-	595	591	628
19 Va. G - Vn. D^b	610	598	601	583	592	603	610	614	606	584	593
Augmented Fourth											
5 Va. A - Vn. D#	606	614	610	600	617	626	630	631	609	626	619
13 Va. C^b - Vn. F	-	-	-	-	-	-	-	-	604	597	605
13 Vc. A^b - Vn. D.	-	-	-	-	-	-	-	-	589	627	605
15 Va. B^{bb} - Vn. E^b	-	-	-	-	-	-	-	-	597	618	595
15 Vc. G^b - Vn. C	-	-	-	-	-	-	-	-	618	637	624
23 Va. G. - Vn. D^b (C#)	609	609	601	601	616	601	614	612	601	585	630

example 29

STATISTICAL ANALYSIS OF HARMONIC TRITONES

	Number of Data	Spread	Extremes	Quartiles	Median	Mean
Diminished fifth	36	61c.	567-628c.	583-602c.	593c.	593c.
Augmented fourth	34	52	585-637	601-619	610	611
2 Va.A - Vn.E^b	11	41	567-608	579-603	595	593
5 Va.A - Vn.D#	11	31	600-631	609-626	617	617
19 Va.G - Vn.D^b	11	31	583-614	592-610	601	599
23 Va.G - Vn.D^b(C#)	11	45	585-630	601-614	609	607

to the sounding tones so that the resolution of the interval is already apparent in its size. In Example 1 above, the 5/7 interval, E - $B^{\flat}$ 583 cents, is part of the harmonic division of the C an octave below its resultant tone. Resolution in one general direction is implied while its enharmonic equivalent, E - $A^{\#}$ 617 cents, is 7/10 in an $F^{\#}$ series and implies resolution in quite another direction. This is a more logical and sensitive solution to the tritone problem than equal temperament and thus it is quite exciting to see in Example 29 that the trios did discriminate between diminished fifths and augmented fourths in performance; the median and mean for 36 diminished fifths was 593 cents, while the median for 34 augmented fourths was 610 cents, and the mean 611 cents. Looking more closely, it can also be seen in Example 28 that of the 6 tritones used only as augmented fourths in m. 13 and 15 of Setting III, 4 were as large or larger than the 7/10 interval, while 8 of the 11 tritones used only as diminished fifths in m. 10 of Settings II and III are essentially the same size as the 5/7 interval. While the diminished fifths in m. 2 are not so consistant as those in m. 10, they are all smaller than their enharmonically equivalent augmented fourths in m. 5 so that the median and mean for the fifths are 595 cents and 593 cents respectively, while those for the fourths are both 617 cents, the size of the 7/10 tritone.

Comparing m. 19 and 23, discussed earlier, it can be seen in Example 28 that in 7 out of the 11 cases the tritones were essentially the same size, while in the remaining 4 the augmented fourth in m. 23, although spelled as a diminished fifth, was significantly larger in size than the diminished fifth in m. 19. In this situation players who differentiate between tritones would have to be guided by context instead of notation. How well a trio was guided by context would have to be judged in terms of how consistently it differentiated between tritones when the notation indicated it should. It seems unlikely that a group which did not make a difference between m. 2 and 5 would make one between m. 19 and 23. In 4 performances Trio A differentiated significantly between m. 2 and 5 once and between m. 19 and 23 once. In each case the second performance was involved. Trio B differentiated significantly between m. 2 and 5 in all 3 of their performances and between m. 19 and 23 twice. While Trio C differentiated between m. 2 and 5 in all 4 of their performances, they did this only once in the case of m. 19 and 23. But 4 discriminations when the notation did not help to 8 when it did is probably a good record and suggests, as do the unequal intervals in fourth chords, that intonation and the auditory image depend more on the way

tones are used than on their notation.

† EDITOR'S NOTE: The second part of Mr. Shackford's article will be published in the next issue of the Journal of Music Theory (April, 1962).

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Philippe de Vitry's Ars Nova:

a Translation

By

The following translation is based on the single complete manuscript source of the Ars nova, Vatican Barberini 307.*1 A few variant readings, however, have been accepted from other sources, especially Paris, B.N. lat. 14741, Paris, B.N. lat. 18514,*2 and parallel passages in Lambert's Tractatus de musica.*3 All deviations from the text of the Vatican manuscript are noted.

The earlier portions of the Ars nova consist of an elliptical and often confused presentation of material plundered from various earlier sources. Many of these sources have been pointed out in an edition of the text prepared by Gilbert Reaney with the collaboration of Andre Gilles and Jean Maillard,*4 and as a rule are not mentioned again here. The present translation, believed to be the first published in English, follows directly.

LEO PLANTINGA

There are three varieties of music: mundana, humana, and instrumentalis. Our concern here is with instrumentalis. Musica instrumentalis is the name for that which has to do with various instruments, such as the cithara, viella, or the monochord, of which we shall deal only with the last. The monochord is an instrument having one string, and produces its concords in three types of modes, namely diatonic, chromatic, and enharmonic. We shall be concerned only with the diatonic.

Diatonic is that which proceeds through two tones and a semitone. It has thirteen species. Of these, the first species is the unison in sound, which is equality in number, as one to one. The second is the octave in sound, which is the duple proportion in numbers, as two to one. The third is the fifth in

sound, which is the sesquialtera proportion in numbers, as three to two. The fourth is the fourth in sound, which is the sesquitertia*5 proportion in numbers, as four to three. The fifth is the whole tone, which is the sesquioctava proportion, as 9:8. The sixth is the minor third, which is the proportion 32:27. The seventh is the major third, which is the proportion 81:64. The eighth is the semitone, which is the proportion 256:243*6. The ninth is the minor sixth, which is the proportion 128:81. The tenth is the major sixth, which is the proportion 54:32. The eleventh is the minor seventh, which is the proportion 16:9. The twelfth is the major seventh, which is the proportion 486:256*7. The thirteenth is the tritone, which is the proportion 729:512.

It must be understood that all inequality proceeds from equality. This is clear if three units, which are recognized as equal, are put together in one place. From this follows the rule: if the first unit is put into the first position, and then the first and second equal units are together put into the second position, and then the unit in the first position, the two equal ones in the second position, and the third equal unit are together put into the third position, there results a series of duples, which is the first species of multiples. This having been thus accomplished, the triples can be derived from the duples, and likewise the others: duples 1, 2, 4; triples 1, 3, 9; quadruples 1, 4, 16.*8 And in every instance, from the first to the last, all inequality proceeds from equality. And let this much about multiples suffice.

And it must be understood that from the duple multiples with terms in reverse order proceed the sesquialtera superparticulars, from the triple [multiples], the sesquitertia, and likewise the other species:

duples with terms reversed 4 2 1*9

sesquialtera 4 6 9

And it must be understood that from the sesquialtera superparticulars with terms in reverse order proceed the superbipartients, which are superpartients, and in the same way from the sesquitertias proceed the supertripartients, and likewise the others:

sesquialtera with terms reversed 9 6 4

superbipartient 9 15 25

And it must be understood that from the sesquialtera superparticulars*10, with their terms in normal order, proceed the duple sesquialteras which are multiple superparticulars, and in the same manner the others:

sesquialtera 4 6 9*11

duple sesquialtera 4 10 25

Likewise, it must be understood that from the superbipartients, which are superpartients, with their terms in normal order, proceed the duple superbipartients, which are multiple superpartients:*12

superbipartient 9 15 25

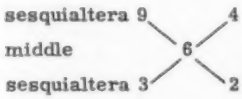
duple superbipartient 9 24 64*13

And it must be understood that if both terms of any proportion are multiplied by the same number, the identical proportion will always result:

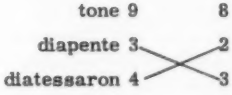
sesquialtera 3 2

also sesquialtera 6*14 4

And it must be understood that if from one proportion you wish to make two, you must multiply the first term by itself, the second term by itself, and then the first term by the second, and you will have the middle term:



And it must be observed that if you wish to find the difference between two proportions, write whatever proportions you wish, but in such a manner that the first term of one will be below the first term of the other, and the second terms likewise. And cross-multiply so that the first term above will go with the second one below, and the second term above with the first below, and you will have this:*15



Concerning the division of the monochord

Now for the division of the monochord. If any string under tension*16 is shortened, its sound will be raised or elevated; *17 and it must be observed that every half of a string sounds similar to its whole. There are two propositions, namely that for similar sounds there are similar signs, for diverse sounds, diverse signs. And it must be understood that two fourths and a tone*18, or a fifth and a fourth make an octave. And it must be understood that a major third and a semitone make a fourth.

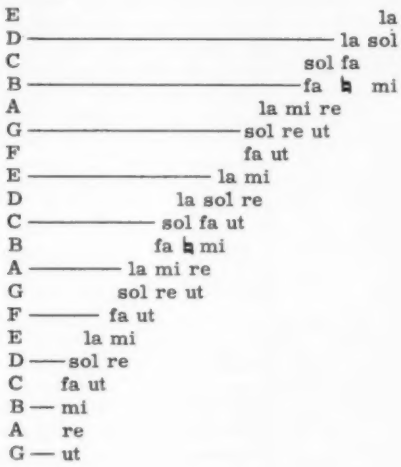
Likewise, it must be understood that, if you wish to construct on a given string all the kinds of musical proportions of the diatonic genus, first a tone must be laid out, then another tone, and thereafter a semitone, from low G to its twelfth, D;*19 then from the octave, G, to its twelfth, D, in the manner described above, then from the fifteenth, G, to the twentieth, and beyond if it be possible for the voice, though this does not conform to our practice; in order that this may be made clearer, it appears in a figure thus:*20

t t s t t s t t s t'st t s t t s t t s t'st t s t t s t'st t s
G a b c d e f G a b♮ c d e f G a b♮ c d e f G a b♮ c d e

Concerning the operation of the monochord

Now what follows concerns the division of the monochord according to its operation. Let there be a string, whose entire length is G, whose half is another G, the half thereof being the third G. Now of the first low G, let three parts be C,*21 whose half is another C, whose half is in turn the third C. Again, of the first C, let three parts yield F, half thereof the second F, and half of that, in turn, the third F. Again, from the low G, two parts are D, and its half another D, and its half the third D. Again, the first D is divided by three, and one third is added to it in the direction of the first G, yielding the first A, whose half is the second A, its half, in turn, being the third A. Again, two parts of the first A give the first E, whose half is the second E, whose half is the third E. Again, the first E is divided by three, and a third part of it is added toward the first G, giving the first B♮, whose half is also B♮, its half, in turn, being the third B♮. Again, from the first F three parts give the first B^{b}, of which the half is the second B^{b}, and its half, in turn, the third B^{b}.

Of these signs, the first eight*22 are called grave, because they represent low song, that is, very low; the next seven are called acute, because they represent high song; the rest indeed, are called superacute, as they are placed above the acute, or because they represent superacute, i.e., very high sounds. Now the previously mentioned seven signs of the monochord, G, A, B, C, D, E, F can be repeated infinitely. *23 But according to our practice, the names of the sounds are six, namely ut, re, mi, fa, sol, la. And they are superimposed on the previously mentioned signs, as ut is placed on any of these, G, C, or F, and the following sounds on the following signs. And in this way the scale is put together. The scale is, then, nothing other than the combination of the signs of the monochord and the sounds; this will appear more clearly in the following figure:



Concerning the parts of music

It must be observed that there are four principal parts of music or of the scale. Thus the first part concerns the signs and the names of the pitches, the second concerns the lines and spaces, the third the proprieties, the fourth the mutations.

Having dealt with signs and pitches, we must now treat the lines and spaces. Line and space, as they are used here, are

nothing other than even and odd. Thus every note on a line is called odd; that which is in a space is called even, or equal. Therefore any sign corresponding with an odd number is on a line, and every one with an even number is in a space. From thence it follows, according to the nature of numbers, that if the first sign be on a line, the following one*24 will be in a space, and in this sense every fourth and eighth tone is opposed to the first; for the eighth, however, the same sign is used. Therefore, if the first pitch is on a line, the other will be in a space and vice versa. And these two*25 rules hold in connection with a staff governed by the same clef.

Concerning the proprieties of music

That which follows concerns the proprieties. A propriety is nothing other than a differentia. There are three kinds, namely B♮, natural, and B♭. Thus B♮, with a square ♮ before it, is said to be a tone from A; B♭, with a round ♭ before it, is said to be a semitone from A; music without any B's is called natural, i.e., without differentiae.*26 From thence comes the rule: whenever ut is on G, it and the following pitches are governed by B♮, and when on C, by the natural state, and when on F, by B♭. Hence the verse:

C naturam dat, F ♭ molle tibi signat,

G quoque ♮ durum facit te esse caniturum.*27

And so much for the proprieties of music.

Concerning the mutations

That which follows concerns the mutations. Now mutation is nothing other than the substitution of one tone for another which has the same sound and the same sign. Thus it follows that at whatever degree a mutation is made, it is necessary that at that degree there be at least two syllables.*28 But at gamut, A re, B mi, and E la there is only one syllable, and thus here there can be no mutation. Nor, similarly, at ♮ mi ♭ fa, because here there are different signs and different sounds. Because they are not put under the same sign, neither do they have the same sound. And for this reason no modulation can be made there, because this would violate the definition. If, in fact, they had the same sound, it would have had to be called B fa mi;*29 and, that all may understand this more clearly, let them look again at the monochord.

It must be observed that where there are two syllables, there are two mutations, as at F fa ut, where one can say fa-ut [or ut-fa]. Now wherever there are three syllables, there are six mutations, as at G sol re ut, and the others. Where there are two syllables, the first is changed to the second and vice versa, and where there are three, the first is changed to the second and vice versa, and the first to the last and vice versa, and the second to the last, and vice versa. And by reason of that, where there are two syllables, their number is not doubled to make four mutations, as three syllables are doubled to make six. Hence the rule that every mutation arriving at ut, re, or mi is called ascending, because it allows for more ascent than descent, and every mutation ending with fa, sol, or la is called descending, because it allows for more descent than ascent.*30 A mutation is undertaken to accommodate ascent or descent, as is clear in the case of C fa ut. For, if at this tone anyone should choose fa, he would be able to ascend as far as the third pitch. But should he wish to go to the fourth pitch, it would be necessary to choose ut of this same C fa ut, which is the mutation from fa to ut.*31 And descent is to be accomplished in the same way. Let this suffice.

Observe that music is the knowledge of accurate singing, or an easy means of achieving perfection in singing.*32 And its name is derived from moys,*33 which is water, and ycos, knowledge, because it was invented according to water. And it has 13 species, namely unison, tone, semitone, etc.

The unison is whatever is accommodated by a single line or single space, wherever it may occur in the scale, that is, under whatever sign of the scale, or at whatever pitch. And its name is derived from unus and sonus, as it has one pitch which is the same both according to figure and sound. Likewise, in another sense unison means the sound of a single pitch, from which no progression is possible, as it must always remain on the same line or in the same space. If, indeed, a progression is made from any pitch to a neighboring one, then at times a tone will result, at times a semitone. However, it must be recognized that the unison is not in itself a consonance, but is the origin of the other consonances; without the unison no consonance is possible.

What is the unison? The unison is the sound by means of which we first begin to sing. And this sound neither ascends nor descends, and according to the ability of the singer, is located either at a high or low pitch. And it is placed in whatever clef that might be required.

Concerning the semitone

The semitone is the interval between two unisons, which in the human voice is incapable of, and will not admit of, division or the interpolation of a middle sound. And between B♮ and C, or E and F, or A and Bb it occurs naturally, i.e., according to the character of the pitches, for between F and E there is a major seventh. But in the case of mi-fa, there is between these names of pitches the same interval as would result from the addition of a sign to any note; this is assumed. Thus there is a semitone between mi and fa.*34

Now semitone is not, as some believe, derived from semi, which means half, because it is less than half of a tone, as appears clearly in the arrangement of the monochord. It is rather derived from semus, meaning imperfect, as an imperfect sound.

The semitone, as Bernardus said, is the sugar and spice of all music, and without it song is corrupted, altered, and destroyed. Now Boethius calculates the proportion of the semitone by solving a certain problem.

Now by means of musica falsa we occasionally make a semitone where there ought not to be one. For in measured music we see that when the tenor*35 of a motet or rondellus has a B, which is sung as B♮, while its discant has the fifth above, it is necessary to sing the high F, or mi by means of musica falsa. For to make a fifth from mi to fa does not produce a good concord, because the interval from that B♮ to the F above is two tones and two semitones, the combination of which by no means makes a concord. And it is necessary that where a fifth separates one sound from another, there must be a good and true consonance.

And from this, clearly, arises the question of what occasions the necessity in regular music of musica falsa, or of false mutation, when nothing governed by rule ought to accept that which is false, but rather the true. To this it is to be answered that false mutation or musica falsa is not useless; indeed it is necessary in order that good sounds may be achieved, and bad ones avoided. For, as has been observed, if we wish to have a fifth, by necessity we must have three tones and a semitone. Thus, if one note is on ♭ fa ♮ mi, and the natural form is indicated, and the other is on the F♮ above, no consonance is formed because there are not three tones and a semitone, but only two tones and two semitones. Nevertheless,

it can be made into a consonance by that which we call musica falsa, that is, when we make a semitone into a tone, or vice versa. This is not, however, false, but only irregular.

Now it must be observed that ♭ fa ♮ mi*36 is not governed by the same principle as the other keys. Instead it is recognized by means of the exceptional addition of the sign ♮ or ♭, just as we sing mi for the F above when it has the sign ♮; or if we attach the sign ♭ to ♭ fa ♮ mi (or to equivalent notes), so that there results the proportion of a tone, the fifth then formed will be consonant. And for this reason musica falsa is sometimes necessary, in order that all consonance or melody, however notated, may be made perfect.

Therefore, you ought to know, as has been indicated, that musica falsa has two signs, namely round ♭ and that other figure ♮. And they have such a property as, in the case of round ♭, to make of a descending semitone a tone, and of an ascending tone,*37 a semitone.*38 And that other figure ♮, effects the reverse; that is, of a descending tone it makes a semitone, and of an ascending semitone, a tone. Nevertheless, in those places where these signs are required, they are, as has been indicated above, not false, but true and necessary. For no motet or rondellus can be sung without them; therefore they are true. Now that which is false is that which is not true; this is therefore not false.*39

Concerning imperfect tempus

Six minims can be put in place of an imperfect tempus. Thus it must be noted that when two semibreves without tails are substituted for an imperfect tempus, they are equal, each being worth three minims, as here:

When there are three, the first is worth three minims, the second two, and the third one, as here:

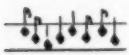
When there are four, the first is minor, the second minimum, the third minor, the fourth minimum, as here:

When there are five, the first three are minimum, the fourth minor, the fifth minimum, as here:

When there are six, all will be equal minims, as here:

And all semibreves without tails must be realized in this

fashion in either perfect or imperfect tempus, while if they have tails, they must be interpreted accordingly. And it must be observed that more than six notes may not be substituted for an imperfect tempus unless semiminims are involved, as here:

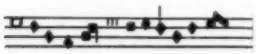


It must be recognized that various names are allotted according to the various values of these semibreves. Thus the semibreve which is worth six minims is called major. The semibreve which is worth five or four, is called semimajor, from semus,*40 that is, imperfect. That which is worth three minims, is called a correct, or true semibreve, while in a general sense, all oblique figures may be called semibreves. That which is worth two minims, is called minor, as has been mentioned earlier; that which is worth only one is called minimum. That which is worth half a minim is called a semiminim. The minims and semiminims, like the other semibreves which have been given names, are designated according to the degree of division. Thus they can be given supplementary names, as the minim is called semiminor, and semiminimum.*41

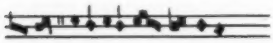
It must be recognized that according to the moderns, just as the minim may be diminished, so may it be augmented. Thus it must be observed that when two minims in a ternary division are put between two semibreves or breves, the second minim is worth two minims, and is called an altered minim. Just so, when two semibreves are put between two ternary*42 breves, the second, which has the value of six minims, is said to be altered, as has been indicated earlier.

Concerning the signs for perfect and imperfect tempus

Now that the breves, semibreves, minims, and semiminims and their values have been treated, we must speak of the signs of perfect and imperfect tempus. It is to be understood that a circular figure is used to designate perfect tempus, because the circle is perfect. But some use three oblique strokes; these indications means the same thing, as here illustrated:*43



In order to explain the partition of each breve*44 into three equal parts in a ternary passage, it must be said that wherever such a circle or three strokes are found without a point of division, perfection is indicated, that is, the tempus is in itself perfect, or properly disposed for division into three equal parts. That this is perfect is thus proven: that which has a beginning, middle, and end is perfect;*45 this is thus composed; therefore, etc. And conversely, that which is without these parts, or one of them, is imperfect; the imperfect is so; therefore, etc. The greater is assumed; the lesser is specified. Now the imperfect tempus is divided into only two semibreves, and thus lacks one of them, etc. Thus, to indicate its imperfection, a semicircle or two strokes are affixed:

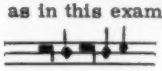
c 

Concerning the variations of perfect and imperfect modus

The signs denoting change of tempus having been treated, it must be pointed out, since modern singers vary both modus and tempus, that there are some songs which are perfect in modus and tempus, others imperfect, others perfect in modus, but not in tempus, others vice versa, and others partly perfect, partly imperfect in modus as well as in tempus.

In order that we may be perfectly familiar with the varieties of modus as well as tempus, we propose to give signs denoting perfect and imperfect modus.*46 But first we must take note of the various modifications of songs.

When there are three tempora, either perfect or imperfect, in any perfection, the modus is said to be perfect; when two, the modus is imperfect. Thus in perfect modus, a long before a long is equal in value to three tempora, unless it is imperfected by a single preceding or following breve. A duplex long is worth six tempora. But in imperfect modus, a simple long is worth two tempora, nor can it ever be worth more, unless a point is added. A duplex [long is worth] four, and can be neither augmented nor diminuted, except by one or two minims, as in this example:



Now in perfect modus, as has been seen, a duplex long is imperfected in two ways, by a single breve, in which case it can be worth only five [tempora], or by two, in which case it will have the value of only four. It can be imperfected as well by minims, in the same way as the duplex imperfect long. Moreover, in perfect modus, the second of two breves between two longs is altered. In imperfect modus, however, it cannot be altered.

Now wherever there are found a number of rests of three tempora represented in single figures, the modus is perfect, as in Orbis orbatus.*47 Where two or more successive rests of two tempora each are found, the modus is imperfect, as in Adesto sancta trinitas.*48

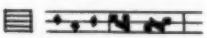
Perfect*49 modus and perfect tempus appear simultaneously in [the motet] which is called Deus iudex fortis. The modus is perfect because every perfection has the value of three tempora. The tempus is perfect because each tempus is divided into three equal semibreves.

Imperfect modus and imperfect tempus appear in Adesto, because here each perfection has the value of two tempora, and because every tempus is divided but into two equal semibreves.

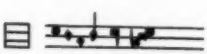
Perfect modus and imperfect tempus appear in Bona condit. Why the modus is perfect and the tempus imperfect has been seen above.*50

Perfect tempus and imperfect modus appear in Marie preconio.*51 Partly perfect and partly imperfect tempus as well as modus appear in Garison.

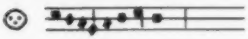
According to some, a square containing three long strokes is used to denote perfect modus, as in this example:



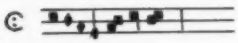
To denote imperfect modus, a square containing two long strokes is used, as in this example:



To denote both perfect modus and perfect tempus, [a circle] containing three strokes is used, the circle indicating perfect tempus, the three strokes, modus, as in this example:



To signify imperfect modus and imperfect tempus, a semicircle containing two strokes is used, as in this example:



But that we may be able to change only the modus, without altering the tempus, we must make use of the sign which, as has been seen, is proper to it alone, namely the square.

Concerning red notes, Chapter I

We must briefly investigate the reasons for using red notes in motets. It must be observed, thus, that they are used principally for two reasons. For instance, red notes are sometimes performed in a mensuration different from that of the black notes, as in Thoma tibi obsequia, where the red notes in the tenor are sung in perfect tempus and imperfect modus, the black notes vice versa. Or they are sometimes used to indicate some sort of reduction of their values, as in the motet In arboris. In the tenor of this motet, three tempora of red notes, but only two of black ones constitute a perfection. Red notes are occasionally used here and there in ballades, rondelli, and motets to indicate that the notes have been rearranged in order that they might be fitted in alternation with those of other perfections, as in Plures errores.*52

[Concerning red notes] Chapter II

Red notes are used in a second way, to indicate that they are sung an octave from the pitch at which they appear, as in Gratia miseri and in the motet called Quant amors. In the tenors of these motets, thus, all the red notes are sung at the octave. Sometimes red notes are used to differentiate the proper chant, i.e., the simple or plainsong, because it is indistinguishable from that which is not plainsong or proper

chant, as in Claerburg. Red notes are sometimes used to indicate that a long before a long is not worth three tempora, or that the second of two breves situated between longs is not altered, as in the tenor of In nova fert animus.*53 Or, they are also used in order that a long before a long may be worth three tempora, and a breve before a breve, three semibreves, as in In arboris. Red notes are also used occasionally to indicate change of tempus and modus, as in the tenor of Garison. In the tenor of this motet the black longs are worth three perfect tempora, the red ones, two imperfect tempora. And occasionally the converse is true, as in the tenor of the motet called Plures errores sunt.

Concerning the names of perfect time

While above we competently treated tempus and prolation according to the division into six or nine minims, in order that we may not appear to have investigated insufficiently the division of the tempus, we wish to deal with it now more precisely. Now it must be understood that there are three kinds of perfect tempus, namely minimum, medium, and major. Franco postulated the minimum tempus. Thus it must be observed that according to Magister Franco, and as has been seen above, the minimum tempus contains but three semibreves, which are indeed so short that they cannot be further divided, unless they be divided by semiminims. And it must be observed that in any song in perfect tempus, where the tempus contains but three semibreves, these semibreves must be performed according to minimum tempus. If there are four, the first two are semiminims, unless otherwise indicated.

Likewise, it must be understood that when two semibreves take the place of this minimum tempus, the first ought to be major, and never the second, unless it is so designated; but we have proven above that according to the Ars vetus, the second ought to be major. The reason for this is that these semibreves in minimum tempus are the same as three minims in major tempus. For when two semibreves are substituted for three minims, the first is worth two minims, the second but one minim, unless there is indication to the contrary, as has been observed above.

Concerning medium perfect tempus

The medium tempus is that which contains three equal semi-

breves, each of which is equal, or ought to be equal to two minims; the medium perfect tempus contains but six minims. And if four notes are substituted for that tempus, two must be minims; if five, four must be made minims; if six, all are equal minims. And any division of these minims results in semiminims, each minim being divided into two semiminims. Therefore, when we see that the tempus is not divided into more than six smaller values, we must sing these values according to medium perfect tempus. We can, however, sing them according to major tempus, when not more than six take the place of a tempus, and these are not differentiated with tails. For if they are differentiated, they must be performed in accordance therewith.

Concerning major perfect tempus

It must be understood that it contains three semibreves, of which each has the value of three minims; and thus the major perfect tempus contains nine minims, and it cannot have more, unless it be divided into semiminims. Thus, when there are (in a single tempus) more than six semibreves, it is necessarily a major perfect tempus; and thus the major perfect tempus is equal to three minimum [perfect] tempora.*54

Concerning minimum imperfect tempus

Now it must be understood that just as there are three kinds of perfect tempus, namely minimum, medium, and major, as has already been observed, there are two kinds of imperfect tempus, namely minimum and major.

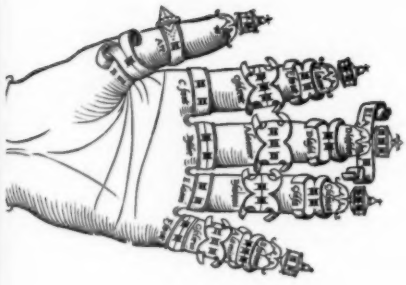
The minimum tempus is that which contains two semibreves, each having the value of two minims; thus the minimum imperfect tempus can only have the value of four minims, unless it be divided into semiminims.

Concerning major imperfect tempus

The major imperfect tempus contains two equal semibreves, each of which has the value of three minims; thus the major imperfect tempus contains six minims. Therefore, when we see that more than four minims take the place of an imperfect tempus, we must sing them according to major imperfect tempus. And thus it is apparent that, just as the perfect

[tempus] is divided into three semibreves, so it also has three manners of performance. The imperfect tempus has two, minimum and major, just as it is divided into two semibreves. And it must be observed that the major imperfect tempus has the same value as the medium*55 perfect tempus.

Here ends the Ars nova of Magister Phillipe de Vetri



references

1 Hereafter cited as V.

2 A list of variants appears in Musica Disciplina XI (1957), unnumbered leaf between p. 12 and p. 13.

3 In Scriptorum de musica medii aevi nova series, E. de Coussemaker, ed., 1864-67 (hereafter cited as CS) I, 251-81.

4 G. Reaney, A. Gilles, J. Maillard, ed., Ars nova magistri Philippi de Vitriaco (hereafter cited as RGM). In: Musica Disciplina X (1956) 13-31.

5 V has sexquialterum.

6 The sentence in V is incomplete: ...super-13-partiens 243 ut 25♯ ad.

7 The sentence in V is incomplete: ...quod est super-230.

8 As noted in RGM, 14, the ultimate source of this passage is Boethius, De institutione arithmetica, bk. I, and De institutione musica, bk. II (G. Freidlein, ed., Leipzig, 1867). In the footnotes to this section and the following one, RGM gives numerous references to the first book of the Speculum musicae of Jacques of Liège, where the same material is systematically treated. If these early sections of the Ars nova are in fact a part of the original treatise, however, the Speculum musicae is almost surely of too late a date to be considered as a source for them.

In De arithmetica, p. 67-68, it is explained that the duple multiples are derived from a series of unities, the triple multiples from the duples, etc., in the following fashion: the first number of the series being derived is equal to the first number of the series from which it is derived; the second number is obtained by adding together the first two numbers of the original series; the third is derived by adding the first number, the double of the second number, and the third number of the original series, thus:

1	1	1	
1	2	4	1, 1+1, 1+2+1
1	3	9	1, 1+2, 1+4+4
1	4	16	1, 1+3, 1+6+9

9 There is an irrelevant interpolation between the two lines of this example in V: sexquialterum 96444.

10 V has superpartienti.

11 V omits the 6.

12 All these demonstrations of the derivation of simple and multiple superparticulars and superpartients are again found in the De arithmetica of Boethius, p. 69-71. The method of derivation is in each case exactly similar to that of deriving the multiples.

13 V has 84 instead of 64.

14 V has Eadem sexquialterum 6 4. Both the text and footnote in RGM are mistaken at this point.

15 This sentence is somewhat confused in V: Et multiplica per crucem, ita quod prima b superior in secundum a inferior, et ultima inferior in prima in prima inferiori, et hababis propositum.

16 V seems to have intensitate rather than in trinitate given by RGM, 16.

17 elongatur.

18 V has semitonio.

19 No attempt has been made to translate or emend the following clause: quod finis dicitur diatessaron propter confusionem differentiarum.

20 V assigns the letter names A-G to the notes without any differentiation of octaves. RGM, 16, has emended the letter names to correspond to the more usual medieval system given by Guido (Guidonis Aretini Micrologus. J. Smits van Waesberghe, ed. (1955) 93-95). Because of the demands of the text, the system appearing in the Vatican manuscript is used here. The translation of this passage has taken into account the reading of Paris, B.N. 18514, as given by the editors of RGM in Musica Disciplina XI (1957) 15.

21 V has: Ergo est autem partes primi G gravis sit [C].

22 Following Paris B.N. 18514, as given in Musica Disciplina XI, unnumbered leaf between p. 12 and p. 13.

23 V has ...et etiam in infinitum potentia (not ponuntur, as given in RGM, 17).

24 reliquum.

25 Adopting the reading of the parallel passage in Lambert, CS I, 255b.

26 Ibid.

27 "C indicates the natural state; F signifies for you Bb, and with G, you must sing Bb."

28 voces. This word is used to mean both tones, and functions or syllables.

29 Following Lambert, loc. cit., 256a. This entire section on mutations is a garbled rehash of the corresponding passage in Lambert.

30 Following Lambert, loc. cit., 256b.

31 Corrections made from ibid.

32 Nota quod musica est scientia veraciter canendi, vel facilis ad canendi perfectionem via. RGM, 20, indicates that this definition also occurs in the Dialogus of Odo (of Cluny?) GS I, 252. Lambert's similar definition of musica harmonica (CSI, 252a) might be a more immediate source for the passage. RGM, 8, further states that this definition "goes back through Isidore to Augustine and even further". Isidore, however, defines music as "peritia modulationis sono cantuque consistens" (Etymologiarum. W. Lindsay, ed. (Oxford, 1911) Lib. III, cap. 15), and Augustine, as "scientia bene modulandi" (De musica. G. Finaert, ed. (Paris, 1947) 24). The latter (op. cit., 24-26) makes it clear that modulandi is not to be equated with canendi.

33 Cf. Johannis Affligemensis De musica cum tonario (J. Smits van Waesberghe, ed. (Amsterdam, 1950) 55) and Hieronymus de Moravia Tractatus de Musica (Cserba, ed. (Regensburg, 1935) 12).

34 The following sentences have been deleted to avoid a contradiction and to bring the passage into agreement with Lambert, loc. cit., 257b-258a: Et dicatur a semis, quod est dimidium, et tonus, quod habens dimidium tonum, ut patet figura. Ita dicatur autem semitonium, quasi imperfectus tonus.

35 V has tenor sive biscantus.

36 V has be molle.

37 V has semitonio.

38 V has tonum.

39 An unrelated and incomprehensible marginal note in V at this point reads:

> Note about the minor third: it is observed that it should contain two tones and a half rather than one and a half, as the major third contains in itself two tones. Therefore, if one be added, it is seen that there are formed two tones and a half, or semitone. This is to be explained by the following rule: when in the formation of numbers, the smaller number precedes the larger (as 1&, 1 before 4), a single unit is subtracted from the larger number; this also applies to the minor third.

For a partial explanation of this passage, cf. Musica Disciplina XI (1957) 21-22, f.n. 5.

40 V has semis in the body of the text, semus in a marginal addition.

41 Following Paris B.N. lat. 7378A in Musica Disciplina X (1956) 48. Cf. also ibid., 37.

42 in modo perfecto.

43 Vel secundum aliquos obliqui apponuntur tractuli tres, et unum est, ut hic. The words uterque and tempus added by RGM, 24, completely alter the meaning of the sentence. The word tempus is inappropriate anyway, as the three lines (which are not distinguishable in the example of RGM, but appear clearly in V) could hardly have the value of one tempus each, since they signify the division of the tempus. V errs only in calling the lines oblique. The parallel passage in the anonymous Compendium artis mensurabilis tam veteris quam novae (CS III, 379a) omits this word.

44 V has semibrevis.

45 Cf. Augustine, op.cit., 70.

46 V has: Et ubi notitiam tam de modo quam temporis variationem habeamus perfectionem, signa aliud modum perfectum aliud modum imperfectum notitiam dare affectamus. Emendations have been made in accordance with Paris B.N. lat. 14741: ut for ubi, denotantia for the second notitiam.

47 V has Orbis orbatur.

48 V has Adesto vetus.

49 V has imperfectus. The following sentence and the reading of Paris B.N. lat. 14741 suggest that perfectus is correct.

50 Following Paris B.N. lat. 14741.

51 Following Paris B.N. lat 738A, Musica Disciplina X (1956) 50.

52 Cf. Musica Disciplina XI (1957) 27, f.n. 10.

53 V has In nova sit animus.

54 V has: et sic maius tempus perfectum tria minima tempora in se continet. The substitution of pro tempore for tempora in RGM, 30, is misleading. The meaning here is clear: the major perfect tempus, which can be divided ultimately into nine minims, is three times as long as the minimum perfect tempus, which, as has been stated above, can be divided into but three semibreves (cf. Franco's Ars cantus mensurabilis, CS I, 122a). There is here a strong suggestion throughout that the minim (or in the case of Franco's minimum perfect tempus, the semibreve) has a constant value.

55 V has maius.

The Source Set and its Aggregate Formations

I wish to acknowledge my indebtedness to Milton Babbitt, whose lectures on combinatoriality delivered at Princeton University in 1952, and whose subsequent articles on the subject led me to initiate the researches which are herein presented.†

As Mr. Babbitt has demonstrated, twelve-tone operations

†The reader's familiarity with "Set Structure as a Compositional Determinant", by Milton Babbitt (Journal of Music Theory, April 1961) as well as "Twelve-Tone Invariants as Compositional Determinants" (The Musical Quarterly, April, 1960) and "Some Aspects of Twelve-Tone Composition" (The Score and I.M.A. Magazine 12, 1955) by the same author, is necessarily assumed. Terms defined in the above articles will not be redefined here.

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(i.e., those which can logically be derived from the twelve-tone system) are well defined and their results can be generalized. Specifics are, as in all music, dependent upon the interpretation of such generalizations.

A knowledge of one's materials and an awareness of their implications would seem to be a basic condition for the intelligent composing of music. The twelve-tone system and the tonal system are the most procedurally sound of all possible pitch collections*1; but the most perfect materials produce parallel results only when they are intelligently handled. It is argued that the partitioning of the twelve-tone set, and conversely, the construction of the set by operation on its subsets is as essential to the orderly communication of ideas indigenous to the twelve-tone system as the equally necessary, and

in some ways analogous procedure of tonal music wherein a collection of elements forming a basic construct — the triad for instance — can be related within or between its classes by an operation — progression by fifth — which produces the total system — less than twelve — and which by extension produces the total number of elements available — equal to twelve. Although the tonal system, whose source constructions are intersecting and whose operations are relatively few, and the twelve-tone system, whose constructions are non-intersecting and whose operations are many, belong to different classes of musical systems, important objectives — the creation of normative, thus predictable, as well as non-normative, thus dramatic procedures which will help to determine as many pitch aspects of the total composition as the composer deems consistent with his compositional intentions — may be realized in each case by exploiting those properties which are peculiar to the system.

In this article I have attempted to present in tabular form all information essential to the calculation of most basic twelve-tone operations. I shall deal in turn with hexachords, tetrachords, trichords, and only summarily with the unequal two-part partitions (1 11), (2 10), (3 9), (4 8), (5 7) of the total set, in each case with a view toward delineating relations within and among partitions. Comments will be confined to the clarification of the tables, and to the general subject of harmony as the result of aggregate-forming combinations. Examples employing number notation are abstract pitch-class compositions and as such are not equivalent to musical compositions.

A musical realization of these "compositions" would necessitate the assignation of at least one element of definition, and would involve a selection — either arbitrary or motivated possibly by other and broader aspects of the total composition — from among all available adjuncts of the pitch structure. A discussion of such procedures clearly exceeds the intention of this paper, which is to present the pitch materials and pitch operations inferable from the following premises: Any ordering of the twelve distinct pitches of the equal-tempered chromatic scale may be regarded as a set 'S' of twelve elements; Any collection of non-intersecting subsets A, B, ... of 'S' which contains all elements of 'S' is a partition of 'S'; An operation on 'S' is an operation on A, B, ... of 'S'. (Operations are defined elsewhere, as explained in the note on the first page.)

The thirty-five source hexachords are presented in a tabular ordering (Table I) selected so as to emphasize the fifteen interval possibilities uniquely associated with each source set and its complement*2. The table divides vertically into four parts on the basis of the number of occurrences of the interval six: 0, 1, 2, or 3 times. Within each part, hexachords are listed according to the number of occurrences of the interval one *3: 5, 4, 3, 2, 1, or 0 times *4, and within these secondary divisions, sets are listed in ascending order of the terms with special emphasis on the last term:

SET	15 intervals 1	2	3	4	5	6
0 1 2 4 5 7	3	3	3	2	3	1
0 1 3 4 5 7	3	3	3	3	2	1
0 1 2 4 5 8	3	2	3	4	2	1
0 1 4 5 6 8	3	2	2	4	3	1

Thus the degree of intervallic similarity — as between set number 1 and number 6 — or dissimilarity — as between set number 1 and number 35 — is immediately apparent.

Given a set whose combinatorial properties are unknown, first reduce it to normal form, then count up the fifteen intervals and consult the table. Once having identified the source set, refer to the four columns labeled Aggregate Transposition Numbers; the combinatorial type and order, as well as the functional transpositions, can easily be discovered. For example, set number 4 (E) is the All-Combinatorial Third Order Set, since three transposition numbers appear in each column. Set number 23, which has a transposition number only in the R column, is obviously one of the eight sets having no special combinatorial properties. (I have labeled these R types.) Henceforth, the all-combinatorial sets will be identified by the code letters A - - - - F; all other sets will be labeled I, RI, P, or R plus the table number.

The formulas associated with Babbitt's four categories of "source hexachords" will hereafter be abbreviated as follows: Define the total set by its two complementary hexachords Pa, b. The six-note content 'a' is distinct from the six-note content 'b' and content ordering is immaterial.

if $b \overset{T}{\sim} a(b=a^t)$ only *5, then $Pa \cup Pa^t$ = A(P type)*6

$\overbrace{0\ 1\ 3\ 4\ 5\ 8}^{a}\ \overbrace{6\ 7\ 9\ 10\ 11\ 2}^{b}$; a^{t6}, 6 7 9 10 11 2=b

if $b \overset{T}{\sim} a$ $(b=a^{+t})$ only, then $Pa \cup Ia^{+t} = A$(I type)

a b
0 1 2 3 4 6 5 7 8 9 10 11; a^{+}, 0 11 10 9 8 6; $a^{+t}11$, 11 10 9 8 7 5=b

if $a \overset{T}{\sim} a^{+}$ $(a=a^{+t})$ [$b=b^{+t}$ follows] only, then $Pa \cup RIb^{+t}$=A(RI type)

a b
0 1 5 6 7 8 2 3 4 9 10 11; a^{+}, 0 11 7 6 5 4; $a^{+t}1$, 1 0 8 7 6 5=a

b^{+}, 10 9 8 3 2 1; $b^{+t}1$, 11 10 9 4 3 2=b

If all three relations hold, the set is all-combinatorial; if none of the relations hold, the set is one of the eight R types. (Remember that $P \cup R=A$ always.)

A majority of first-order hexachords themselves contain non-intersecting subsets to which the above equations are applicable. (See Table I, IV or IVa.)

(0 1 2) (3 4 5) $b=a^{t}3$, $a=a^{+t}2$, $b=a^{+t}5$

(0 1 4) (2 3 6) $b=a^{t}2$

Note that in sets of higher order, $(a^{t}=b)$ times the maximum number of all-combinatorial subsets equals 12 or a multiple of 12 (i.e., all prime transposition numbers taken together form a symmetrical construct which splits the octave equally).

(0 1 2) (6 7 8) or (0 2 7) (6 8 1) $a^{t}6=b$, 6x2=12, (0 6)

(0 1) (4 5) (8 9) or (0 5) (4 9) (8 1) etc. $a^{t}4=b$, 4x3=12, (0 4 8)

(0) (2) (4) (6) (8) (10) $a^{t}2=b$, 2x6=12, (0 2 4 6 8 10)

Since the hexachord can itself be partitioned equally by 2, (3^{2}), (3 3) or, conversely, derived by systematic operation (priming, inverting, following, etc.) upon one (column 1 under trichords) or two (column 2) trichords, the generators which fulfill these requirements are included in Table I. Thus our table reveals all basic information concerning the source set's intervallic structure, combinatorial type and order, the specific transposition numbers for aggregate formation (secondary set numbers are easily determined), as well as the trichordal mosaics contained therein.

The aggregate produced by the simultaneity of two permutationally related set forms is not necessarily ordered. But the product of set union would be at least one new set type. This "derived harmonic set," whose combinatoriality depends at least upon content placement of the parts with respect to one

table I

SOURCE HEXACHORDS

No.	Set	15 Intervals 1 2 3 4 5 6	P	AGGREGATE TRANSPOSITION NUMBERS I	R	RI	TRICHORDS (zero is omitted to save space)▸ One Generator	Two Generators◆
A1	012345	5 4 3 2 1 0	6	11	11	6	12, 13, 14, 24	12-15, 13-14, 13-25
B2	023457	3 4 3 2 3 0	6	1	1	6	13, 15, 24, 25	13-27, 13-37, 14-25
3	013458	3 2 3 4 3 0	6	1	1	6	13, 15, 24, 25	13-27, 13-37, 14-25
E4	014589	3 0 3 0 3 0	2, 6, 10	3, 7, 11	3, 7, 11	2, 6, 10	15	12-15, 13-14, 13-37, 14-25, 14-37, 15-27, 24-48, 25-37
C5	024579	1 4 3 2 5 0	6	3	3	6	14, 15, 37, 48	
							24, 25, 27, 37	13-25, 15-27, 25-37
6	012346	4 4 3 2 1 1		11	11		14	12-13, 12-16, 12-26, 13-24, 13-25, 13-26, 13-36, 14-15, 24-25
7	012356	4 3 3 2 2 1		11	11		(25)	
8	012456	4 3 3 2 2 1			3	3	(12, 14, 15, 25)	(12-15, 12-48, 13-16, 13-26, 13-37, 14-15, 14-26, 15-24, 24-27)
9	012357	3 4 2 3 3 1		11	11		(15)	12-24, 12-27, 13-16, 13-25, 13-26, 14-37, 15-25, 24-27, 25-26
10	012457	3 3 2 3 3 1			11			
11	013457	3 3 3 3 2 1			2		(13)	
12	012458	3 2 3 4 2 1		11	11			12-14, 13-15, 13-37, 14-26, 14-36, 14-48, 15-16, 15-26, 24-37, 25-37
13	014568	3 2 2 4 3 1		3	3			12-15, 13-14, 14-16, 14-26, 13-24, 15-37, 15-48, 16-37, 25-37, 26-37
14	014578	3 1 3 4 3 1			6		(14, 36, 37)	
15	013468	2 3 3 3 3 1			2		(25)	
16	013568	2 3 3 2 4 1			4		(13, 36)	
17	013578	2 3 2 3 4 1			6	2	(13, 15, 27, 37)	(12-24, 14-25, 15-24, 15-37, 16-25, 16-37, 25-26, 26-37, 27-48)
18	014579	2 2 3 4 3 1		3	3			13-14, 14-24, 14-25, 15-16, 15-25, 15-26, 26-37, 27-37, 36-37, 37-48
19	023579	1 4 3 2 4 1		1	1		37	13-24, 13-25, 15-37, 16-27, 24-25, 25-26, 25-27, 25-36, 26-27
20	012367	4 3 2 2 3 2		11	11		26	12-14, 12-16, 13-15, 13-16, 14-16, 15-37, 16-36, 25-27
21	013567	4 2 1 2 4 2			4	3	(12), 15, 16, 26, (27)	(12-15, 13-16, 14-16, 15-27, 16-25, 16-37)
22	013467	3 2 4 2 2 2			5	2	(13), 14, (15, 25), 26, (36)	(12-36, 13-16, 13-37, 14-16, 15-36, 25-37)
23	012366	3 3 2 2 3 2			11		(16)	
24	013478	3 2 2 3 3 2			11		(16)	
25	012578	3 2 2 2 4 2		11	11		26	12-13, 14-15, 13-25, 15-25, 16-27, 16-36, 16-37, 27-37
26	012468	3 1 4 2 2 2		11	11			12-24, 13-26, 14-26, 15-24, 15-48, 16-26, 24-27, 25-26, 26-37
27	023468	2 4 2 4 1 2		1	1			12-26, 13-24, 13-26, 14-48, 15-26, 16-26, 24-25, 24-37, 26-36
28	023568	2 3 4 2 2 2			4	4	(13, 15, 25, 36)	12-36, 13-25, 15-26), 14-37, (16-26), 24-36, (25-26, 27-36)
29	013469	2 3 3 2 2 2		11	11		15, 26	13-14, 13-25, 13-36, 14-36, 14-37, 25-36, 25-37, 36-37
30	013569	2 2 4 3 2 2			4	2	(14, 16, 36, 37)	13-14, 13-25, 14-26, 15-36, 16-26, 24-36, 25-37, 26-37, 36-48)
31	013679	2 2 4 2 3 2			7	2	13, 15, 25), 26, (36), 37	13-14, 14-25, 15-36, 16-25, 27-36)
32	013579	1 4 2 4 2 2		11	11			13-24, 14-24, 15-26, 15-36, 24-25, 25-36, 26-37, 37-48
D33	012678	4 0 2 4 3 2	5, 9	5, 11	5, 9		12, 15, 16, 27	16-26
34	013679	2 2 4 2 2 3		5, 11	5, 11		13, 14, 25, 37	16-26, 16-36, 26-36
F35	024681	0 0 6 0 3 0	1, 3, 5, 7, 9, 11	1, 3, 5, 7, 9, 11	1, 3, 5, 7, 9, 11	1, 3, 5, 7, 9, 11	24, 26, 48	

+ Bracketed trichords in R and RI hexachords indicate that the trichord will not generate the complementary form.

◆ Consult Table IV5 for type R hexachords.

another, may have profound meaning, if only locally, through the formation and maintenance of, as well as progression through, hexachordal harmonies*7 distinct from those which could be produced by ordered fragmentation of the original set. Yet these two seemingly opposed processes — fragmentation and derivation*8 — might ultimately reveal themselves to be but different expressions of a single concept. (See Example 3.)

By treating this "product of a specific combination" as a new set, capable of aggregate formation, we can greatly extend the possibilities of the original set. In Example 1, a type "A" set combines with its retrograde to form two aggregates of type I12.

Example 1

P	0	4	11	3	1	2	5	6	9	8	10	7
R	7	10	8	9	6	5	2	1	3	11	4	0
	I12						I12					

(Note that a direct combination was selected and in this case was absolutely necessary since the remaining partitions do not generate sets with special properties.

	R14										R14					
	0	4	11	3	1	2				or	0	4	11	3	1	2
	7	10			8	9	6	5			7				10	8	9	6	5

For the most part this discussion of resultant harmony assumes point-against-point combinations the product of which, depending upon the number of voices employed, will be twelve dichords, trichords, tetrachords, hexachords, or twelve-note chords. Actual compositional results, though dependent upon these "background harmonies," are always a matter of individual choice.)

Since type I12 is consistently obtained, we can now exploit its special combinatorial properties and multiply by 2.

Example 2

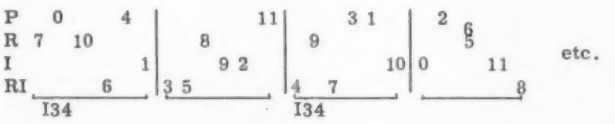
P	0	4	11	3	1	2	5	6	9	8	10	7
R	7	10	8	9	6	5	2	1	3	11	4	0
I	1	9	2	10	0	11	8	7	4	5	3	6
RI	6	3	5	4	7	8	11	0	10	2	9	1

To effect this four-part combination it is necessary to employ two set forms (I_1 and RI_6) which are not members of the sub-

group of order 8 (see this journal: Vol.V, p.75 and 79) associated with P_0. It will be demonstrated that under certain conditions all four forms are members of the subgroup of order "n" associated with P_0. The set of Example 3 is one such instance ("4 group").

Before carrying out the next extension to eight parts, order characteristics, not necessarily reflecting a compositional selection of vertical textures and attack points, will be imposed upon the combination.

Example 3



Under these conditions the exact opposite compositional operation might be assumed, namely the construction of linear sets, 0 4 11 3 1 2 etc., by fragmentation of one set, 7 0 10 6 4 1 3 5 8 9 2 11. Here the resultant harmonic hexachord, having assumed order characteristics, emerges either as the basic set, or as a new construct possibly capable of challenging to some extent the primacy of the basic set. But regardless of our interpretation, the most significant aspect of the above example is the interdependency of vertical and horizontal pitch events.

In order to extend the structure to eight parts the following unequal part partitions are necessary:

P (2 1 2 1) R (2 1 1 2) I (1 2 1 2) RI(1 2 2 1)

I (2 1 2 1) RI(2 1 1 2) P (1 2 1 2) R (1 2 2 1)

Example 4

P	0	4	11		3	1	2		
R	7	10	8		9		6	5	
I	1		9	2	10		0	11	
RI	6		3	5	4	7	8		etc.
P	2		6	1	5		3	4	
R	9		0	10	11	8	7		
I	3	11	4		0	2	1		
RI	8	5	7		6		9	10	

This process might logically extend to as many simultaneous set forms as there are methods (i.e., registral or timbral delineation) needed to define it.

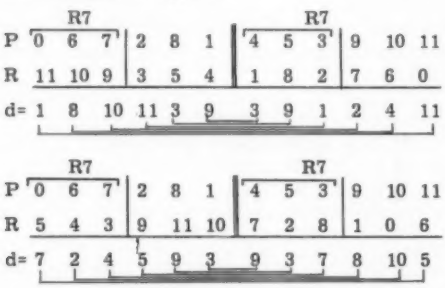
A concern for harmony in the small (i.e., within the derived harmonic hexachords) leads to an examination of the verticalities — dichords, trichords, etc. — which result from direct combinations of two or more set forms*9. Assuming a non-degenerate basic set, each of the four classes of two-part combinations*10 (P ∪ P, P ∪ R, P ∪ I, P ∪ RI) guarantees unique vertical associations and thus unique compositional results.

If P∪ P=A obviously a single interval is maintained throughout and, depending upon combinatorial order, in one of the following pitch-pitch patterns: $\begin{matrix}0 & 6\\ 6 & 0\end{matrix}$; $\begin{pmatrix}0 & 6\\ 3 & 9\end{pmatrix}\rightarrow\begin{pmatrix}3 & 9\\ 6 & 0\end{pmatrix}$, $\begin{pmatrix}3 & 9\\ 0 & 6\end{pmatrix}\rightarrow\begin{pmatrix}6 & 0\\ 3 & 9\end{pmatrix}$ etc.

When P ∪ R=A the resultant intervals are predictable only with respect to a given set ordering and may be all odd, all even, or a mixture of both. Intervals produced by the union P(0-5)∪ R(0-5) will be complemented through retrogression in P(6-11) ∪R(6-11), but unless $P_{(11)}=R_{(0)}$ (Example 5a) the pitch-pitch relations $\frac{x}{y}\rightarrow\frac{y}{x}$ will not be maintained within the combination. (Example 5b). In sets of higher order, where more than one combination by retrogression is possible within the sub-group of order "n," the intervals obtained by a given combination are altered in the latter cases (second order set, t=6; third order set, t=4 or 8; etc.), but the ratio of intervals odd to even will be preserved*11:

Example 5

Linear Sets of Type D



The complete combination table for Example 5 is given below.

$P_0 \cup R_{11}$	$P_3 \cup R_2$	$P_6 \cup R_5$	$P_9 \cup R_8$
$P_3 \cup R_8$	$P_6 \cup R_{11}$	$P_9 \cup R_2$	$P_0 \cup R_5$

An examination of the three remaining transpositions of these combinations within the subgroup of order 16 associated with our type "D" set shows that:

1) each interval stated within a given combination is transposed in turn by the symmetric tetrachord 0369 and therefore eight (two basic combinations times four transpositions each) combinations associate with P_0, eight with P_1, and eight with P_2; taken together they yield all transpositions of all intervals obtainable within the combination class.

2) The twenty-four pitch-pitch relations distributed within the eight combinations occur twice in the form $\frac{x}{y}$ and twice in the form $\frac{y}{x}$.

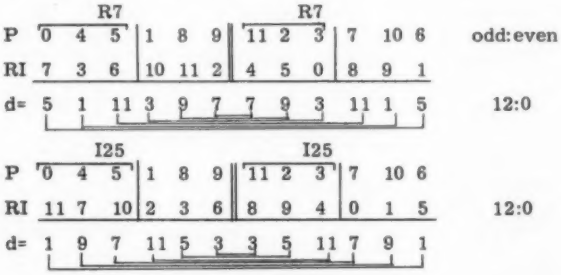
3) Each of the four transpositions of a combination at a given interval preserves the order of occurrence of the intervals as well as the derived harmonic aggregates.

4) The interval ratio is maintained throughout.

When $P \cup RI = A$ the results are similar to combination by retrogression, except that the intervals obtained by $P(0\text{-}5) \cup RI(0\text{-}5) = A$ are retrograded in $P(6\text{-}11) \cup RI(6\text{-}11) = A$.

Example 6

Linear Sets of Type E



	R7						R7						
P	0	4	5	1	8	9	11	2	3	7	10	6	12:0
RI	3	11	2	6	7	10	0	1	8	4	5	9	
d=	9	5	3	7	1	11	11	1	7	3	5	9	

When P ∪ I=A, all six odd intervals occur twice, a tritone distant, such that $\frac{x}{y} \rightarrow \frac{y}{x}$ for all values of $\frac{x}{y}$ within a single combination. In first order all-combinatorial sets, $\frac{x}{y} \rightarrow \frac{x}{y}$ when the combination is moved to the remaining transposition level within the subgroup and in sets of higher order there will be more possibilities within the circuit. The combination table for the type "E" set of Example 6 is given below.

Example 7

	series x y	series r s	series t u
"di" #1	$P_0 \cup I_3$ $P_6 \cup I_9$	$P_2 \cup I_5$ $P_8 \cup I_{11}$	$P_4 \cup I_7$ $P_{10} \cup I_1$
#2	$P_2 \cup I_1$ $P_8 \cup I_7$	$P_4 \cup I_3$ $P_{10} \cup I_9$	$P_6 \cup I_5$ $P_0 \cup I_{11}$
#3	$P_4 \cup I_{11}$ $P_{10} \cup I_5$	$P_6 \cup I_1$ $P_0 \cup I_7$	$P_8 \cup I_3$ $P_2 \cup I_9$

All six combinations within a single vertical column preserve pitch-pitch relations, while all six combinations within a horizontal column preserve the order of intervallic occurrence ("di"). Note that each basic interval of combination together with its transposition (t=6) is equivalent to one of the all-combinatorial tetrachords $\begin{pmatrix} 0 & 6 \\ 3 & 9 \end{pmatrix}$ = f, $\begin{pmatrix} 2 & 8 \\ 1 & 7 \end{pmatrix}$ = e, $\begin{pmatrix} 4 & 10 \\ 11 & 5 \end{pmatrix}$ = e.

In the next example one member of each of the four classes of combination is applied to a type "A" set. Each combination yields different derived combinatorialities as well as different resultant interval collections.

Example 8

		R15				R15					D				I34			
8:4	P	0	4	11	1	3	2	7	5	10	6	8	9	0:12 P	0	4	11	1	3	2	7	5	10	6	8
	R	9	8	6	10	5	7	2	3	1	11	4	0	P	6	10	5	7	9	8	1	11	4	0	2
		3	8	5	3	10	7	5	2	9	7	4	9		6	6	6	6	6	6	6	6	6	6	6

	R16			R16					RI21			RI30			
4:8	P	0 4 11	1 3 2	7 5 10	6 8 9	12:0	P	0 4 11	1 3 2	7 5 10	6 8 9
	RI	6 7 9	5 10 8	1 0 2	4 11 3		I	9 5 10	8 6 7	2 4 11	3 1 0
		6 9 2	8 5 6	6 5 8	2 9 6			3 11 1	5 9 7	5 1 11	3 7 9

The criteria for the measurement of harmonic distance have been discussed by Babbitt*12. Set transpositions and set forms existing within the subgroup maintain, irrespective of order, the hexachord content of P_0, while transposition out of the collection brings about "modulation" in that a greater or lesser intersection of content obtains. The "modulation" is either distant or near depending upon the degree of content intersection.

Transpositions and forms of derived harmonic hexachords produced by the combination of related linear forms may similarly be measured. When a combination produces a single derived set of minimally semi-combinatorial type, there will be at least one other combination, whose class is dependent upon combinatorial type, which maintains content identification of the derived hexachords. Thus in Example 1, $(P_0 \cup R_7)$ type I12 is consistently obtained and the other form is $(I_1 \cup RI_6)$. But here transposition of the combination within the subgroup of its vertical forms causes a transposition outside the subgroup of its linear forms. If, however, the basic set is derived from a single trichord, transposition to a subgroup member will insure content preservation within linear hexachords as well as within derived hexachords, and transposition of a combination outside the subgroup of its linear forms will similarly affect the derived harmonic hexachords; thus the criteria for the measurement of harmonic distance can be applied simultaneously to both dimensions. The next example exploits this property in order to obtain secondary sets and aggregates at all levels of the structure*13. The modulatory "following" $P_3 \cup I_2$ produces a linear intersection of the three notes 6 7 8 in one form, 0 1 2 in the other (0 1 2 6 7 8 is the type D set), and a corresponding intersection of the three notes 2 4 6 between adjacent derived sets (taken with 8 10 0 from the non-adjacent sets a type F set is produced).

Example 9

	3 1		2 4 5		10 11 8		6 7 9	R	3 1 0		2 5 4		11 10 8		7 9 6	P	3 6 4		5 7 8
type B				type B				type B				type B				type B			
1 8 10	9 7 6		1 0 3		5 4 2	RI	8 10 11		9 6 7		0 1 3		4 2 5	I	2 11 1		0 10 9		

Thus, a degree of harmonic change clearly exists both within the collection of combinations associated with the subgroup and within all classes of combination included. The transposition of combinations to levels within the subgroup can, as in Example 9, preserve content of both linear and vertical sets, while transposition to levels outside the subgroup effects strong shifts in harmonic stability since the harmonic hexachords will similarly be transposed and since pitch-pitch relations (except in the case of P ∪ P) will be altered. Furthermore, each combination class produces different resultant combinatorialities (see Example 8) except when the basic set is trichordally derived (Example 9), in which case, although the combinations are still differentiated by the dichords, P ∪ I = P ∪ RI and P ∪ R = P ∪ P with respect to type.

Of the twenty-nine source tetrachords*14 (see Table II) seven, those which exclude the interval '4', are independently combinatorial. The twenty-two possible combinations of these seven sets are presented (Table IIa) in the form of tetrachordally derived, tetrachordally combinatorial source sets of twelve notes. Each total set is in turn capable of aggregate (and secondary set) formation.

Example 10

a	,	0	3	2	1	#9	P	,	0	3	2	1	4	6	5	7	8	10	11	9
a+	,	7	4	5	6		RI	,	6	4	5	7	8	10	9	11	2	1	0	3
a	,	9	10	11	8		P	,	8	11	10	9	0	2	1	3	4	6	7	5
		A																		

Since tetrachordal combinatoriality depends upon the exclusion of the interval '4', Table II divides into four parts on the basis of the number of occurrences of this interval, 0, 1, 2, or 3 times. The arrangement within parts follows the procedure of Table I.

There are twelve independent content orderings of 's', the semi-combinatorial source tetrachord (P type)*15. Of the four all-combinatorial sets of first order ('a', 'b', 'c', 'd'), the first three have eight independent orderings each, of which four are degenerate (P=RI). Set 'd' has six independent orderings of which none is degenerate. The second order set 'e' with two transpositions for content preservation (0, 6), has but six orderings of which all are degenerate, while the third order set 'f' with four transpositions is equivalent at all content levels (0, 3, 6, 9) and there are but three forms — all

tables

II

SOURCE TETRACHORDS

No.	Set	δ Intervals 1 2 3 4 5 6
a1	0123	3 2 1 0 0 0
d2	0127	2 1 0 0 2 1
e3	0167	2 0 0 0 2 2
b4	0235	1 2 2 0 1 0
a5	0136	1 1 2 0 1 1
c6	0257	0 2 1 0 3 0
f7	0369	0 0 4 0 0 2
8	0124	2 2 1 1 0 0
9	0134	2 1 2 1 0 0
10	0125	2 1 1 1 1 0
11	0136	2 1 0 1 1 1
12	0156	2 0 0 1 2 1
13	0135	1 2 1 1 1 0
• 14	0146	1 1 1 1 1 1
15	0236	1 1 2 1 0 1
• 16	0137	1 1 1 1 1 1
17	0147	1 0 2 1 1 1
18	0157	1 1 0 1 2 1
19	0237	1 1 1 1 2 0
20	0247	0 2 1 1 2 0
21	0238	0 1 2 1 1 1
22	0358	0 1 1 1 2 0
23	0145	2 0 1 2 1 0
24	0347	1 0 2 2 1 0
25	0158	1 0 1 2 2 0
26	0246	0 3 0 2 0 1
27	0258	0 2 0 2 0 2
28	0148	1 0 1 3 1 0
29	0248	0 2 0 3 0 1

IIa

THE 22 TETRACHORDALLY DERIVED SOURCE MOSAICS

No.	Set	Aggregate Nos. P	Tr. Nos. I	R	RI
1	$a_0\ a_4\ a_9$	4, 8			
2	$a_0\ a_8\ b_3$	4, 8			
3	$a_0\ b_3\ c_9$	4, 8			
4	$a_0\ d_5\ e_4$	4, 8			
5	$a_0\ a_7\ a^*_5$	4, 8			8
6	$a_0\ c_2\ a^*_{11}$	4, 8			8
7	$a_0\ d_5\ a^*_8$	4, 8			8
8	$a_0\ f_2\ a^*_{10}$	4, 8			8
9	$a_0\ a_4\ a_8$	4, 8			0
10	$a_0\ b_3\ c_4$	4, 8	3		
11	$a_0\ d_5\ a_3$	4, 8			0
12	$a_0\ e_4\ a_8$	4, 8	3	3	0
13	$b_0\ b_4\ b_8$	4, 8			0
14	$b_0\ f_4\ b_8$	4, 8	5	5	0
15	$c_0\ c_4\ c_8$	4, 8			0
16	$c_0\ d_5\ c_1$	4, 8			0
17	$c_0\ c_3\ c_8$	4, 8	7	7	0
18	$d_0\ d_4\ d_8$	4, 8			0
19	$d_0\ b_6\ d_3$	4, 8			0
20	$e_0\ e_4\ e_8$	4, 8; 2, 10			0; 6
21	$e_0\ f_2\ e_3$	4, 8; 2, 10	7; 1	7; 1	0; 6
22	$f_0\ f_4\ f_8$	4, 8; 2, 10 0, 7; 1, 11	0	0	0

Transposition numbers for I, R, and RI are computed by adding the column number to the P aggregate numbers.

If, in sets of two or three different tetrachords, the odd member is in an outer position only P P P and P I I combinations are available.

degenerate: thus greater combinatorial flexibility is accompanied by an increasing reduction and degeneracy of the forms.

The product of tetrachordal combinations is a quartet of trichords whose members are not necessarily distinct. Table III documents the following discussion of quartets produced by the combination of related forms of a single tetrachord:

1. Prime related combinations generate four transpositions of the same trichord: 0 4 8, 1 5 9, 2 6 10, 3 7 11. Sets of higher order have more possibilities.

2. Each ordering of a given tetrachord combined with its inversionally related forms yields the same vertical pitch relations.

Example 11

0	1	2	3		0	1	3	2
11	10	9	8	j	11	10	8	9
7	6	5	4	=	7	6	4	5

3. Under P∪R∪R, P∪RI∪RI, or P∪R∪I, and in the case of non-degenerate forms therein, the results depend upon class of tetrachord ordering (see Table III; 0123, 0213, 1032, 1302 belong to one class, 0132, 0312 belong to another class).

3a. Under P∪R∪R and P∪RI∪RI, all orderings of a given tetrachord which share the same quartet share the same vertical pitch relations.

Example 12

P	0		2	5		3	j	P	0		5		2	3	j	P	0		5		3	2	j	P	0		2	5		3	j	P	0		5		2	3	j	P	0		5		3	2
P	4		6	9		7	=	P	4		9		6	7	=	P	4		9		7	6	≠	P	4		6	9		7	=	P	4		9		6	7	=	P	4		9		7	6
R	11	1	10	8		R	11	10	1	8		RI	11	10	8	1		RI	10	8	11	1		RI	10	11	8	1		R	10	11	1	8													

3b. Under P∪I∪R, and in the case of non-degenerate forms thereof, orderings which share the same quartet share pitch relations if those orderings are members of the same class.

Example 13

0		1	3	2		0	3	1	2			0		2	3	1		0	3	2	1															
11	10	8	9	j	11	8	10	9	≠	11	9	8	10	j	11	8	9	10																		
6		7	5	4		6	5	7	4			5		7	6	4		5	6	7	4															

Table III indicates that each available combination class

table

III

TETRACHORD ORDERINGS	TRICHORDAL QUARTET UNDER:				
	P-P-P column 1	P.I.I column 2	P-R-R column 3	P.RI.RI column 4	P.I.R column 5
a †0123, 0213, 1032, 1302	(48)4	(15, 37)2	see col. 2	see col. 1	see col. 2
0132, 0312	"	"	(26)4	(37)4	(16, 25, 27, 36)
0231, 0321	"	"	(37)4	(26)4	(16, 25, 27, 36)
b †0235, 0325, 2053, 2503	(48)4	(14, 37)2	see col. 2	see col. 1	see col. 2
0253, 0523	"	"	(14)	(26)4	(13, 16, 16, 25)
0352, 0532	"	"	(26)4	(15)4	(13, 16, 16, 25)
c †0257, 0527, 2075, 2705	(48)4	(14, 15)2	see col. 2	see col. 1	see col. 2
0275, 0725	"	"	(14)4	(26)4	(12, 13, 16, 36)
0572, 0752	"	"	(26)4	(14)4	(13, 13, 16, 36)
d 0567, 0657, 5076, 5706	(48)4	(26, 48)2	(14, 37)2	(14, 37)2	(13, 14, 26, 37)/ (14, 27, 36, 37)
0576, 5067	"	"	(24, 26)2	(24, 46)2	(24, 24, 29, 26)
e †0167, 0617, 0671, 0761	(48)4/(24)4	(14)4/(37)4/(13, 25)2	see col. 2	see col. 1	see col. 2
0176, 0716	" "	" " "	see col. 2	see col. 1	see col. 2
f †0369, 0396, 0639	(48)4/(27)4/ (24)4/(15)4/ (12)4	(24)4/(15)4/(24, 48)2/ (13, 27)2/(15, 15, 27, 27)/ (12, 12, 15, 15)	see col. 2	see col. 1	see col. 2
s 0135, 0316, 1053, 1503	(48)4		(24, 26)2		
0143, 0613, 1036, 1306	"		(14, 15)2		
0361, 0631, 3016, 3106	"		(15, 37)2		

The symbol † indicates a degenerate form.
The symbol (15) indicates the trichord 015.

(degenerate forms reduce the number of such classes) guarantees unique resultant harmonies and thus, as is the case with hexachords, unique compositional results.

The following table further summarizes Table III.

	12	13	14	15	16	24	25	26	27	36	37	48
a				x	x		x	x	x	x	x	x
b		x	x	x	x		x	x			x	x
c	x	x	x	x	x			x		x		x
d	x		x			x		x		x	x	x
e		x	x			x	x				x	x
f	x		x	x	x	x			x			x
s				x		x		x			x	x

A consideration of the remaining equal part partition (2^2) relates resultant harmony to source hexachords. Under a given operation (combination), all orderings of the same tetrachord which share dyad content within their parts share resultant hexachordal combinatorialities.

Example 14

3	1	10	8	1	3	8	10	3	1	8	10	3	10	1	8	1	8	3	10	3	10	8	1
11	9	6	4	9	11	4	6	11	9	4	6	11	6	9	4	9	4	11	6	11	6	4	9
0	2	5	7	2	0	7	5	0	2	7	5	0	5	2	7	2	7	0	5	0	5	7	2
I6				I6				I6				I25				I25				I25			

In the case of P∪I∪R, or in sets of higher order, there is a choice of transposition and therefore a choice of resultant harmony.

Example 15

3	2	9	8	3	2	9	8	9	8	3	2	9	8	3	2
11	10	5	4	5	4	11	10	5	4	11	10	11	10	5	4
0	1	6	7	0	1	6	7	0	1	6	7	0	1	6	7
A				A				E				E			

When disjunct dyads of a given tetrachord ordering are intervallically degenerate. resultant hexachords are maintained under P∪I∪I, P∪R∪R, and P∪I∪R, and are altered under P∪P∪P and P∪RI∪RI*16.

Example 16

I	1	11	8	10	R	11	1	10	8	R	11	1	10	8	P	8	10	1	11	RI	10	8	11	1
I	9	7	4	6	R	7	9	6	4	I	9	7	4	6	P	4	6	9	7	RI	6	4	7	9
P	0	2	5	3	P	0	2	5	3	P	0	2	5	3	P	0	2	5	3	P	0	2	5	3
		I9					I9					I9					F					F		

Three-part tetrachordal combinations, subject to the properties of their resultant sets of three or six notes, may be extended to six or twelve parts. Tables III and IV can be used to determine such extensions.

Example 17

P	0	2	5	3				0	2	5	3				0	2	5	3					
P	4	6	9	7				4	6	9	7				4	6	9	7					
RI	10	8	11	1				10	8	11	1				10	8	11	1					
		F		F		RI	5	3	0	2				5	3	0	2						
						RI	9	7	4	6				9	7	4	6						
						I	11	1	10	8				11	1	10	8						
							RI	21	RI	21		R	3	5	2	0							
												R	7	9	6	4							
												I	1	11	8	10							
												I	2	0	3	5							
												I	6	4	7	9							
												RI	8	10	1	11							

Every twelve-tone set, considered in terms of its equal three part partition (4^3), is capable of at least one three-part "oblique combination"*17 ($P \cup R \cup P$); greater combinatorial flexibility depends upon symmetries within and between outer tetrachords. If parts are defined as x, y, and z, then

x	y	z
	z	y	x
		x	y	z

If $X \overset{T}{\sim} X^+$ only, then

P			
	R		
		P/I	

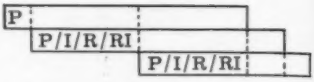
If $Z \overset{T}{\sim} Z^+$ only, then

P			
	R/RI		
		P	

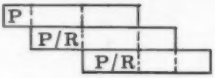
If $X \overset{T}{\sim} X^+$, $Z \overset{T}{\sim} Z^+$ only, then

P			
	R/RI		
		P/I	

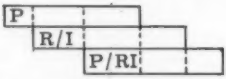
If $X \overset{T}{\sim} X^+$, $Z \overset{T}{\sim} Z^+$, $X \overset{T}{\sim} Z$, then



If $X \overset{T}{\sim} Z$ only, then



If $Z \overset{T}{\sim} X^+$ only, then



The twenty-two combinatorial source sets (Table IIa) allow "direct" as well as "oblique" combinations:

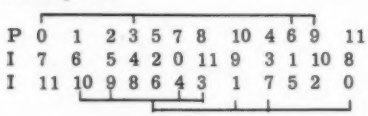
P	0 - 3	4 - 7	8 - 11
11	0 - 3	4 - 7	8 - 11
111	0 - 3	4 - 7	8 - 11

Prime relations are always possible; retrograde and inversion relations are possible only with those sets which retain unit content on inversion, retrogression, or both (sets #10, #12, #14, #17, and #21). The following combination classes exist: P∪P∪P=A, P∪P∪I=A, P∪P∪R=A, P∪P∪RI=A, P∪I∪R=A, and may be applied to sets which contain transposition numbers in the proper columns of Table IIa. (If prime related or inversionally related combinations are employed, or if outer tetrachords are permutations of the same tetrachord ordering, Table III can be used to calculate the harmonic results.) In general, the trichords produced by three-part direct combinations of the total set are extensions of two-part events. Prime related combinations produce a single trichord whose vertical pitch rotations

x y z
y→z→x
z x y

are four times transposed. When P∪RI∪P (see Example 10a.) trichords of complementary order number are related by transposition (i.e., trichords of Ha are retrograded in Hb). In the following inversionally related combination, dichords are fixed for inversionally related forms; and each trichord is transposed by the interval 6, inverted at the interval 3, and then transposed by 6 : 0 3 6 9. The four permutationally related trichords form a total set.

Example 18



The content within tetrachords, as well as the form of the "followings," may be ordered, and those orderings permuted such that various types of hexachordal combinatoriality result. For example, the mosaic abc yields hexachordal type B, I9, and R10; bca yields A, RI22, I6, and R7; cab yields C, I19, I9, RI31, and R16. The set of Example 19 is hexachordally, tetrachordally, and trichordally combinatorial.

Example 19

	a				b			c						
	0	2	1	11	4	9	6	7	5	3	8	10		
				B						B				
P	0	2	1	11	4	9	6	7	5	3	8	10	x = 0 → y = 7	
I	7	5	6	8	3	10	1	0	2	4	11	9	y = 7 → x = 0	
P	0	2	1	11	4	9	6	7	5	3	8	10	x	0 w 10
I	7	5	6	8	3	10	1	0	2	4	11	9	y = 7 → z = 9	
R	10	8	3	5	7	6	9	4	11	1	2	0	w = 10 → x = 0	
RI	9	11	4	2	0	1	10	3	8	6	5	7	z	9 y 7
P	0	2	1	11	4	9	6	7	5	3	8	10	o, o, o^+, o^+, = set	
I	9	7	8	10	5	0	3	2	4	6	1	11	p, p, p^+, p^+, = set	
P	4	6	5	3	8	1	10	11	9	7	0	2	q, q, q^+, q^+, = set	
	o	p	o^+	p^+	q	q^+	o	o^+	p^+	q^+	p	q		

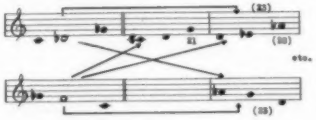
The tables of series IV provide all information essential to the calculation of the aggregate and derived set formations which employ from one to four distinct trichords. Table IVa presents data, summarized in Table IV, concerning the hexachords which can be derived by operation upon a single trichord. For example, the set 013 followed by $(013)^{t6}$ forms a type I34 hexachord; the complementary hexachord may be derived therefrom (mosaic aa/a^+a^+)*18 or may employ new generators (mosaic aa/bb). Mosaics of the type ab/ab, ab/ac or ab/cd are the subject of Table IVb. For sets of the type aa/bc consult both tables*19. (Tables IVa and IVb are read as follows:

The symbol 'a' after a set number indicates the complementary form of the set number as listed in Table I (i.e., 29 is the table number, 29a is the complement).

table IV

SOURCE TRICHORDS

Trichord	P. tr. nos.	L. tr. nos.	Derived Sets of 12 Hexachord Type All-comb.	I	RI	P	Derived Sets of 6 Hexachord Type R	RI
012	0, 3, 6, 9	2, 5, 8, 11	A, A, D					(8), (21)
013	0, 6	5, 11	A, B	34			(11)	(17), (28)
							(16)	(22), (31)
014	0, 2	9, 11	E	6	22		(14a)	(9), (30a)
	0, 10	5, 7	E	6	32			
	0, 6	9, 3	A, E	34				
015	0, 6	3, 9	B, D, E					(8), (17)
	0, 2	11, 9	E	9	21			
	0, 10	7, 9	E	9	21			
	0, 3, 6, 9		D			3		
016	0, 3	8, 11	D	29	21		(23)	(22a), (30a)
	0, 9	8, 5	D	29	21		(24a)	(28), (31)
024	0, 1, 6, 7	4, 5, 10, 11	A, C, F					
	0, 5, 6, 11	4, 9, 10, 3	A, C, F					
	0, 3, 6, 9	4, 7, 10, 1	B, B, F					
025	0, 6	3, 9	B, C	34			(7)	(8), (28)
							(15a)	(22a), (31a)
026	0, 1	10, 11	F	29	31			
	0, 11	10, 9	F	29	31			
	0, 5	10, 3	F	25	22			
	0, 7	10, 5	F	25	22			
	0, 3	10, 1	F	29	21			
	0, 9	10, 7	F	29	21			
027	0, 3, 6, 9	2, 5, 8, 11	C, C, D					(17), (21a)
036							(14)	(22), (30a)
							(16)	(28), (31a)
037	0, 2	1, 11	E	19	31		(14a)	(17), (30a)
	0, 10	9, 11	E	19	31			
	0, 6	5, 11	C, E	34				
048	0, 1, 2, 3	8, 9, 10, 11	E, E, F					
	4, 5, 6, 7	0, 1, 2, 3	E, E, F					
	8, 9, 10, 11	4, 5, 6, 7	E, E, F					

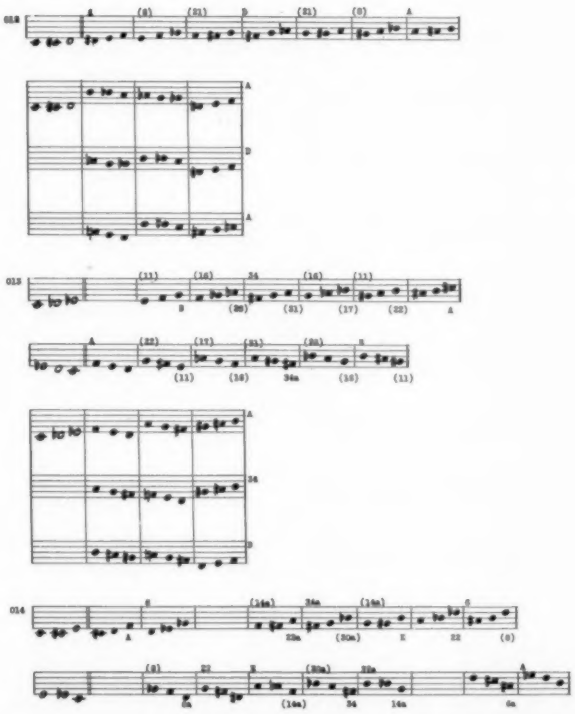
All-combinatorial trichords are underlined.

When set numbers are bracketed, the complementary hexachord is derived from different generators.

table

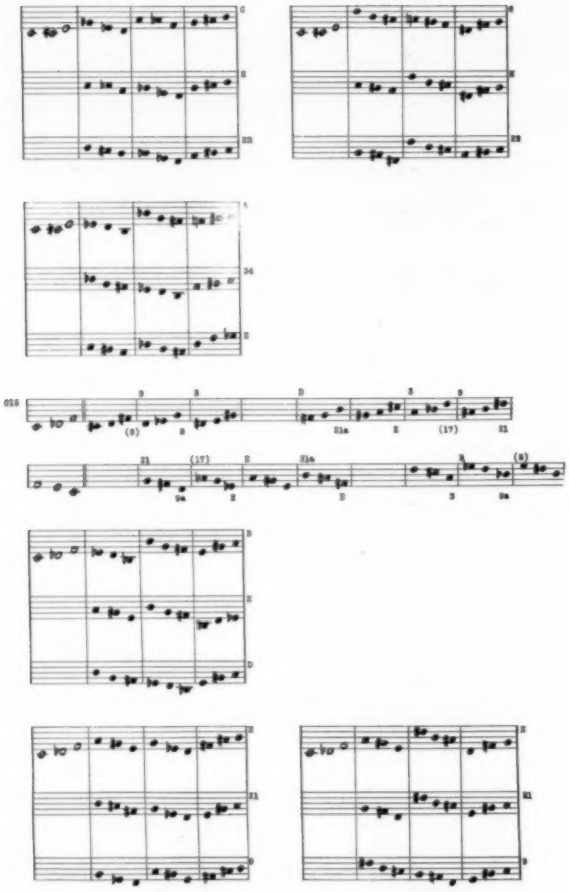
IVa

TRICHORDAL MOSAICS (SINGLE GENERATOR)

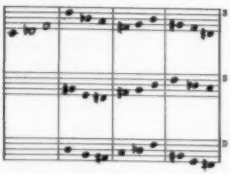
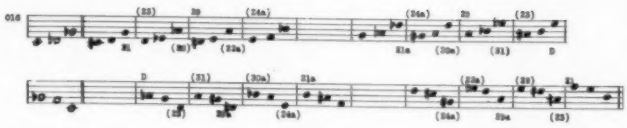
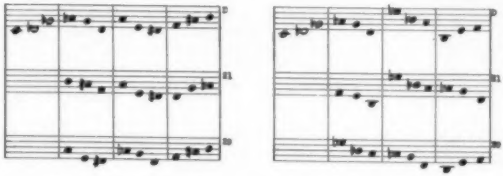
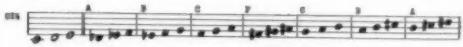
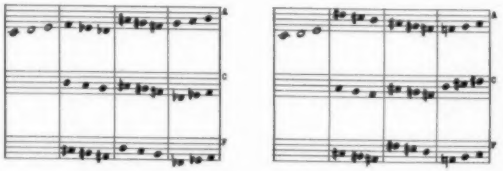
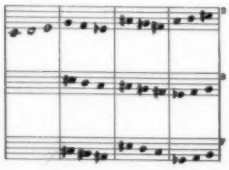



table

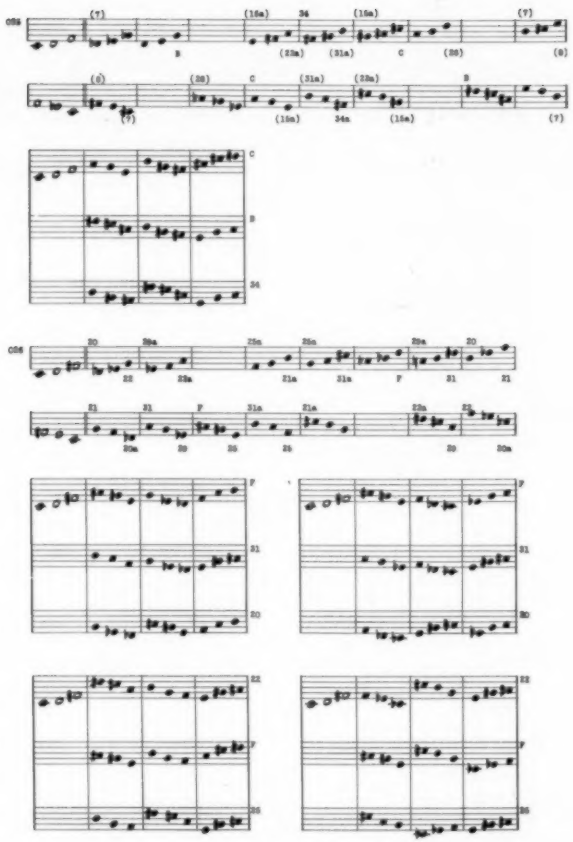
IVa

table

IVa



table

IVa

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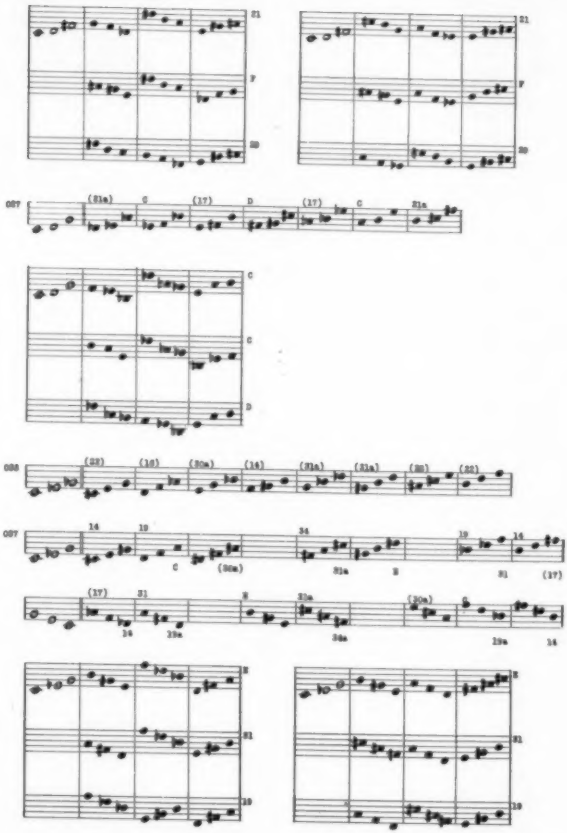
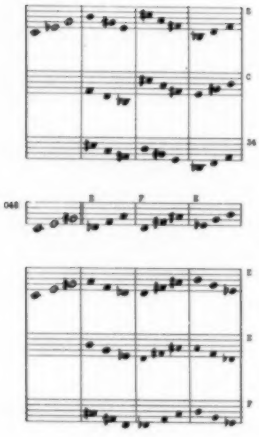
IVa

table

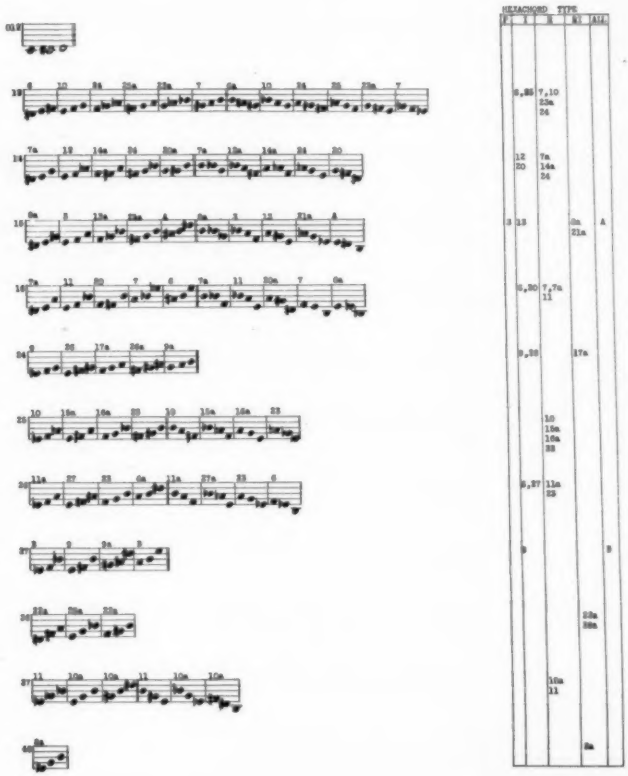
IVa

table

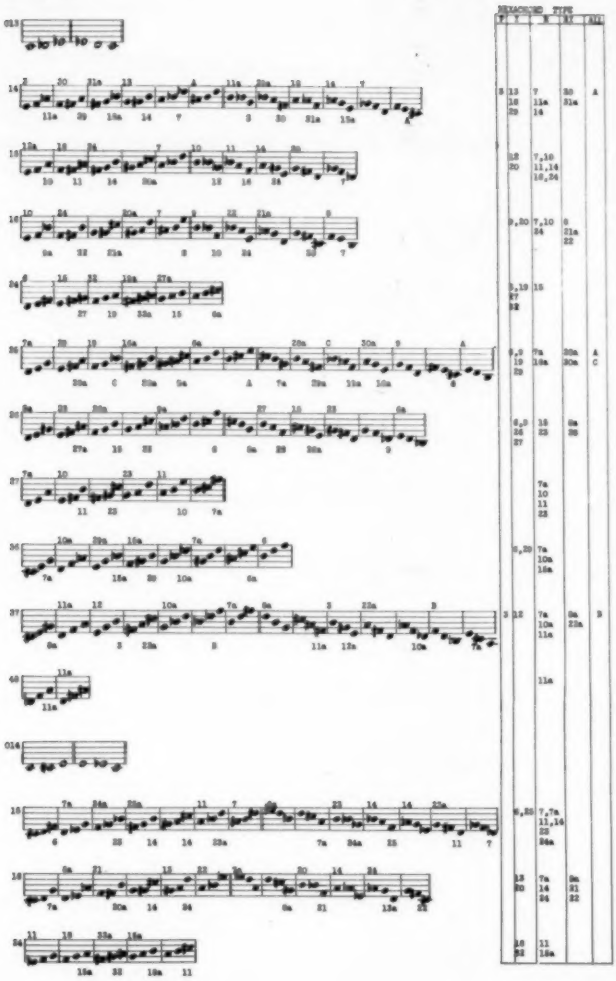
IVb

TRICHORDAL MOSAICS (MORE THAN ONE GENERATOR)



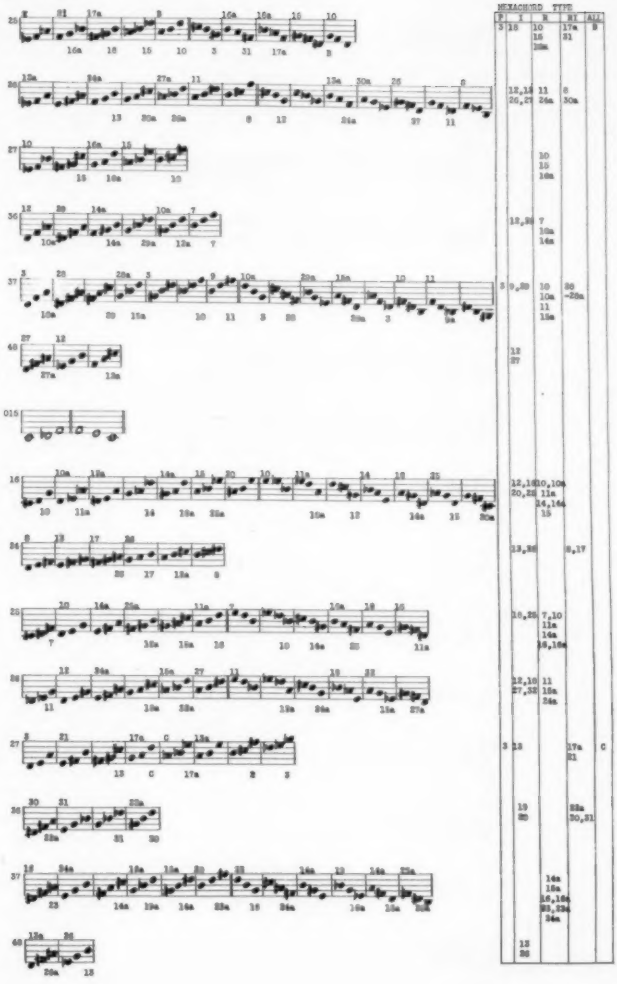
table

IVb

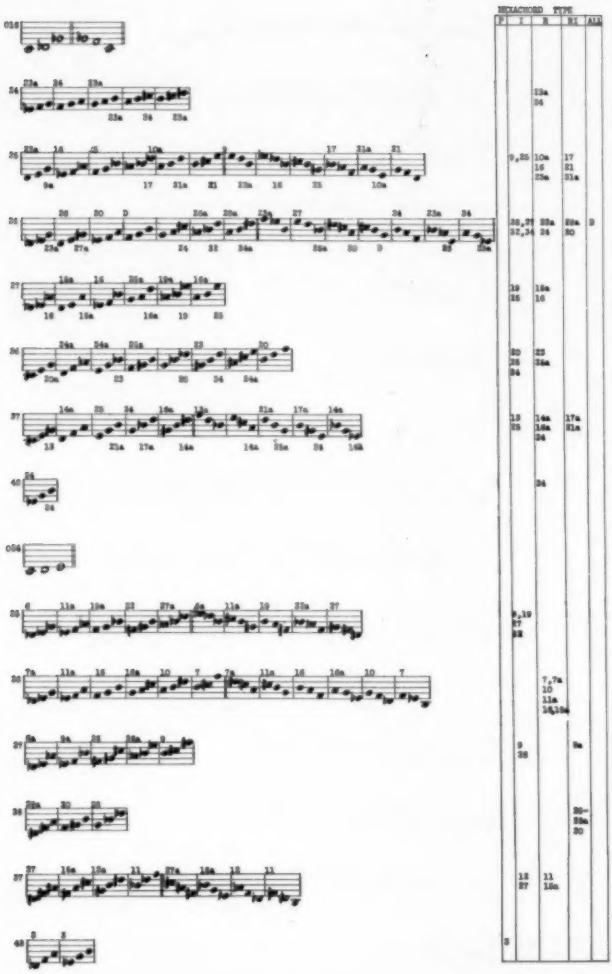
table

IVb

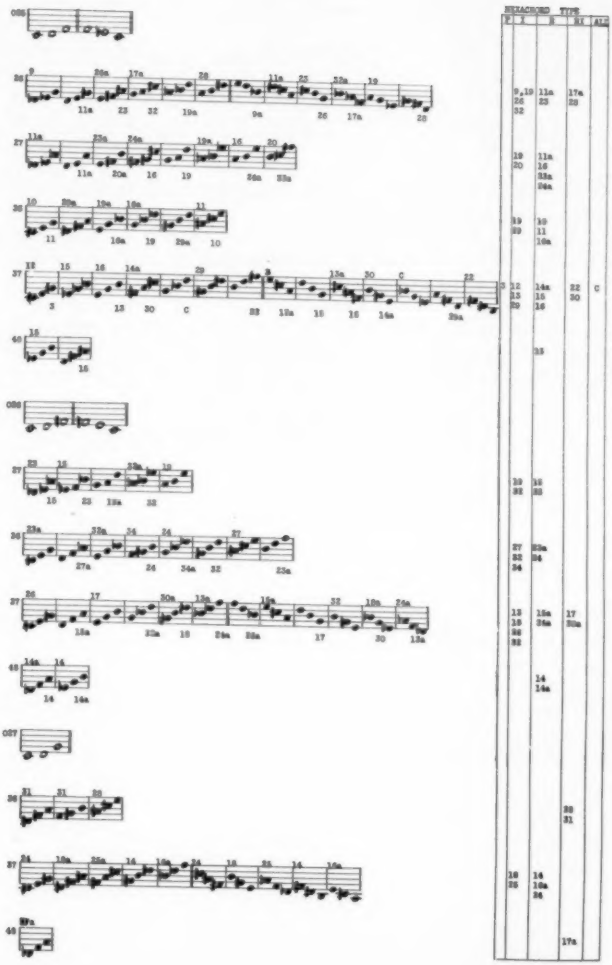
table

IVb

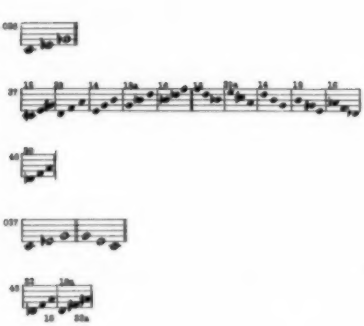



table

IVb

table

IVb

Aggregate-forming combinations of trichords yield a tetrachord trio. Complete results for combinations of related forms of a single ordered trichord are listed in Tables V and VI. Since the tables are cross-referenced, one can begin either with the quartet of trichords (V) or with the trio of tetrachords (VI). A few examples should illustrate the possibilities:

P	9	10	11	×		I	11	10	9	+×		I	11	10	7	+			
P	6	7	8	×		P	6	7	8	×		I	9	8	5	+			
P	3	4	5	×		I	5	4	3	+×		P	2	3	6	+			
P	0	1	2	×		P	0	1	2	×		P	0	1	4	+			
	f	f	f				e	f	e				b	c	a				

The basic set considered in terms of its equal-part partitions might be accompanied by independent permutation of its parts; this need not produce additional permutationally related forms of the total set. A discussion of the partition (3^4) should serve as a model for all possible equal-part partitions of twelve.

Each trichord must be capable of a two-part combination which will produce common hexachordal combinatorialities. Example 20 employs a basic set of one generator.

Example 20

	RI21							RI21					
P	0	1	5	RI	4	8	9	I	11	10	6	R	7 3 2
RI	2	6	7	R	3	11	10	P	4	5	9	I	1 0 8

I	9	8	4	P	0	1	5	R	7	3	2	RI 6 10 11
R	3	11	10	I	7	6	2	RI	8	0	1	P 4 5 9

For one, two, or more generators the procedure is as follows: From Table IV, and IVa select trichords which yield common combinatorialities by the same operation. For example, 013 and 014 both yield type A by inversion, and type I34 by retrogression. Table IVb lists the possible source mosaics 013, 014tr.=H. After a few tests the proper mosaic for type A or I34 is discovered.

table V

TRICHORD/TETRACHORD INTERSECTION

	Pc Pc Pc P Tetr.	Pc Pc Pc I Tetr.	Pc Pc I o I Tetr.	Pc Pc I o I Tetr.	Pc Pc Ro R Tetr.	Pc Ro Ro R Tetr.	Pc Pc Ro R Tetr.	Pc Pc Ro Ro RI Tetr.	Pc Ro Ro Ro RI Tetr.	Pc Ro I o Ro RI Tetr.
012	0 3 6 9 nf	0 3 8 11 24f4b9 0 6 5 11 efe	5 6 11 17f17	see Col. II	see Col. III	0 8 11 2 15f15	see Col. I	see Col. I	see Col. II
120	1 4 7 10 nf	1 4 7 10 f24z41	4 7 10 f17z71	1 4 6 9 23f22 1 7 5 9 27f27	1 3 6 9 21f21		1 4 8 11 222z22	1 5 8 11 211z21	1 3 7 11 299z99 1 5 10 8 c24z5
	"	"	"	"	"	"	"	"	"	"
023	0 6 5 11 efe	0 6 5 11 efe					0 6 2 8 27f27		0 9 5 8 24f24 0 9 11 2 bfb
130	1 7 4 10 fee	1 7 4 10 fee					1 7 5 11 27e27		1 6 4 11 ccc 1 6 10 5 25e25
201	3 9 2 8 eef	3 9 2 8 eef					3 9 4 10 eee		3 7 2 10 25e25 3 7 8 4 23e23
014	0 2 9 11 bca	0 2 9 11 bca					0 2 5 7 ccc		0 6 9 7 aca 0 6 11 5 ecc
	0 4 9 3 fee	0 4 9 3 fee					0 6 5 11 eee		0 10 9 11 aaa 0 10 3 5 cec
140	1 3 8 10 cab	1 3 8 10 cab					1 3 9 11 26a26		1 2 8 11 aaa 1 2 10 9 23a24
	1 7 8 2 eef	1 7 8 2 eef					1 7 9 3 27e27		1 6 8 3 ccc 1 6 2 8 25e25
401	4 8 5 7 abc	4 8 5 7 abc					4 8 9 10 26b26		4 3 5 10 dbd 4 3 7 8 23b25
	4 10 5 11 efe	4 10 5 11 efe					4 10 8 2 27f27		4 7 11 5 bfb 4 7 11 8 24f24
015	0 3 6 9 nf			0 3 11 2 9f9 0 6 8 2 27f27	0 8 11 2 15f15				
025							0 8 10 4 27e27		0 11 3 4 23e23 0 11 9 10 aaa
036							0 2 6 4 26c26		0 7 11 4 24c24 0 7 9 6 ece
146	1 4 7 10 fff			1 4 6 9 23f22 1 7 5 9 27f27	1 3 6 9 21f21				
137	1 7 2 8 eef						1 7 3 9 27e27		1 6 2 9 25e25 1 6 8 3 ccc
248	1 3 10 8 cab						1 3 11 9 26a26		1 2 10 9 23a24 1 2 8 11 aaa
569	5 8 11 2 fff			5 8 7 10 abc 5 11 4 10 efe	5 4 7 10 efe				
	5 11 10 4 efe						5 11 2 8 fff		5 7 10 8 bfb 5 7 4 2 bfb

table V

| | | |

table V

037	0 3 6 9 fff	0 3 8 11 2424	0 3 6 11 171770	0 1 10 4 18f18	0 3 1 4 9249	0 10 1 4 151715	0 10 5 4 252489
	"	0 6 5 11 eef	-	"	0 6 10 4 27e27	"	0 10 11 1 a2483
							0 1 5 4 23e23
370	0 3 6 11 fff	2 5 8 0 3474	2 3 6 9 17f17	see Col. III	see Col. I	see Col. I	0 1 11 10 aaa
		3 6 1 0 dfe	see Col. II				see Col. II
037		0 10 11 0 aab			0 10 4 2 36e38		0 5 11 2 aca
							0 5 9 4 25e24
		0 6 11 5 eef			0 6 4 10 27e27		0 1 11 10 aaa
370							0 1 5 4 23e23
		3 1 8 8 cbb			3 1 11 9 29b29		3 10 8 0 dbd
							3 10 8 11 25b23
		3 8 8 2 afe			3 8 11 5 27f27		3 8 8 8 bfb
703							3 8 8 11 24f24
		7 5 4 2 bac			7 5 6 6 aaa		7 1 4 8 aaa
							7 1 2 8 aab
		7 1 4 10 fee			7 1 8 2 eee		7 5 4 2 cac
							7 5 10 8 aaa
040	0 1 3 6 aaa		0 2 3 5 bab	see Col. III	see Col. I	see Col. I	see Col. II
*			0 1 3 6 aab				
			0 1 8 7 cdd				
		0 1 6 3 aaa	0 1 8 7 cde				
		0 3 1 6 aba					
		0 3 6 8 aab					
	0 1 3 3 aaa	0 3 1 3 aab	0 3 6 6 fac				
		0 1 8 3 aaa	0 1 8 3 aab				
	0 3 3 5 bbb	0 3 6 8 fbb					
	0 3 5 7 ccc	0 3 8 7 cac	0 3 7 8 cdc				
	0 1 3 7 ddd	0 1 3 7 ddb					
	0 1 8 7 eee	0 1 8 7 eef					
	0 3 6 8 fff						

* Entries for 040 are summarized and are listed in the order of Table IIa.

table VI

TETRACHORD/TRICHORD INTERSECTION

	012	013	014	015	016	024	025	026	027	037	048
a a a											1
a a b											1
a b c											1
a d a						1,2					1
a a a			2	2		2		2,3		3	1
a c a			1	1		2		2,3		1	1
a d a											1
a f a						1,2					1
a a a								3		3	1
a b c			1,2,3	1,2,3				3		1,2,3	1
a d a											1
a a a			1			1,2	1	1,2,3	1	1,3	1
b b b				3				3			1
b f b		1	2	3	1	1,2	3	1,2,3		2	1
c c c			1					3			1
c d c											1
c e c	3	3	1,3	3		1,2		1,2,3		3	1
d d d											1
d b d			3	3		2		2,3		2	1
e e e		3	1			1,2	2			3	1
e f e	1,2	1,2,3	1,2,3	1,2,3		1,2	1,2,3		1,2	1,2,3	1
f f f	1,2			1,2,3		1,2			1,2		1
9 f 9				1					1		
15 f 15									1		
17 e 17						3					
17 f 17	1,2					3			1,2		
21 f 21	2										
22 f 22	2			2							
23 a 23						2		2			
23 e 23		3		1		2	1,2	1	1	1	
24 b 24						2		2			
24 f 24	1,2	1	3		1,2,3	2	3	1	1,2	2	
25 c 25								2			
25 e 25	3	2,3	2	2		2	2	1			
26 a 26			2	2							
26 b 26			3							2	
26 c 26				1						1	
27 e 27	2	2	2	1,2			1			1	
27 f 27	2	1	3	1,2			3		1	2	
12 e 17						2					
23 a 24			2	2	3	2		2	1	1	
23 b 25			3			2		2		2	
24 c 25	2			1	2	3		2		1	
9 24 9					3				1		
15 17 15									1		
21 17 21	2										
22 24 22	2				2						
39 24 39	2				2,3				1		

Trichord column numbers refer to rotation, first, second, or third (ie., 014, 140, or 401)

◆ Entries for 048 are summarized and are listed in the order of Table IIa.

Example 21

	P3						
P	0	1	3	P	4	5	8
		I34				I34	
R	9	7	6	R	2	11	10
		P3					

The next step is to operate upon the combination:

Example 22

P	0	1	3	4	5	8
R	9	7	6	2	11	10
I	5	4	2	1	0	9
RI	8	10	11	3	6	7

The second hexachord is then derived and order characteristics may be imposed.

One last example illustrates trichordal permutation at each level of a set containing four distinct members.

Example 23

	B											
0	1	3	10	2	5	7	8	11	4	6	9
9	7	6	11	8	4	5	2	1	3	0	10
	I34										
5	4	2	7	3	0	10	9	6	1	11	8
8	10	11	6	9	1	0	3	4	2	5	7

The possible all-combinatorial source mosaics of two generators are summarized below:

Trichords	Source mosaic	Combinatorial type and operation necessary to produce ⟶	Combinatoriality common to both trichords
012, 015	012 10 11 3	A ∪ tA	D
012, 027	012 8 11 3	B ∪ tB/B^{+}	D
013, 014	013 10 11 2	A ∪ tA	I34
013, 025	013 5 8 10	C ∪ tC	I34
013, 037	013 10 2 5	B ∪ tB	I34
014, 025	014 9 11 2	B ∪ tB	I34
015, 027	015 8 10 3	C ∪ tC	D
016, 026	016 5 7 11	D ∪ tD	I29
025, 037	025 3 7 10	C ∪ tC	I34

The combinatorial properties of subsets suggest the additional possibility of 4, 6, 8 and 12-part aggregate-forming combinations of the total set. If the forms employed are to be members of the subgroup of order "n", some regulation of the basic set is necessary.

Type I semi-combinatorial sets, whose inversionally related forms produce aggregates at the hexachord level, may be extended to form aggregates at the trichord level. Trichords are defined as a, b, c, and d. If $c \overset{I}{\sim} a^+$, $d \overset{I}{\sim} b^+$ (and in some cases $b \overset{I}{\sim} a$), then P∪ I∪ R∪ RI. The source mosaics are ab/a^+b^+, aa/a^+a^+.

Example 24

P	0	1	3	9	8	5	7	4	6	10	2	11	P_0/RI_8
I	7	6	4	10	11	2	0	3	1	9	5	8	I_7/R_{11}
RI	8	5	9	1	3	0	2	11	10	4	6	7	RI_8/P_0
R	11	2	10	6	4	7	5	8	9	3	1	0	R_{11}/I_7

Inversionally related forms maintain dyads; the first six vertical tetrachords are retrograded in the second half; and total "verticalization" of the '4' group of P_0 is accomplished. (Continuation of the "canon" by either of the recommended "followings" produces secondary sets.)

Type RI semi-combinatorial sets may similarly be extended when $b \overset{I}{\sim} a^+$, $d \overset{I}{\sim} c^+$ (in some cases $c \overset{I}{\sim} a$). The source mosaics are aa^+/bb^+ or aa^+/aa^+.

Example 25

P	0	2	6	5	11	9	3	7	1	10	8	4
RI	7	3	1	10	4	8	2	0	6	5	9	11
I	11	9	5	6	0	2	8	4	10	1	3	7
R	4	8	10	1	7	3	9	11	5	6	2	0

In this special case involving a single trichord, vertical tetrachords are fixed.

The type P semi-combinatorial set combines as follows: P∪ P∪ R∪ R. The mosaics are ab/ab or aa/aa.

All-combinatorial sets derived by operation upon one or two source trichords, regardless of mosaic ordering (i.e., ab/ba or ab/ab) or method of construction (i.e., aa^+/bb^+ or ab/ab), will have at least two related two-part combinations which are

capable of extension. The P∪P or P∪R combinations of a set whose source mosaic is ab/ab are capable in the former case of extension by retrogression or in the latter by transposition (P∪I will extend only in the special case $a=a^{+t}$ and/or $b=b^{+t}$) while each basic combination of the mosaic $aa^{+}/a^{+}a$ is capable of extension in a number of ways. Thus four-part combinations depend upon trichord characteristics as well as upon trichord content and placement.

The reader may determine for himself the total number of such source mosaics or may refer to Tables I, IV, IVa and IVb. (Remember that the combinations presently under discussion differ significantly from the four and eight-part extensions illustrated in section one of this paper.)

Example 26

P	0	3	1	2	4	5	8	11	10	9	6	7	I	5	2	4	3	1	0	etc.
I	11	8	10	9	7	6	3	0	1	2	5	4	P	6	9	7	8	10	11	
RI	4	5	2	1	0	3	6	7	9	10	8	11	R	1	0	3	4	5	2	
R	7	6	9	10	11	8	5	4	2	1	3	0	RI	10	11	8	7	6	9	

Note that unlike semi-combinatorial combinations, new forms continue the "canon" and produce secondary sets.

Eight part combinations, given a set whose combinatorially related forms represent a subgroup of at least order '8', are calculated as follows:

1) P∪I∪R∪RI extends by t=6 (P∪I∪P∪I extends by retrogression), when complementary hexachords are ordered such that $'b'='a^{+t}'$ for any ordering of 'a'. 0 1 2 3 4 5 / 11 10 9 8 7 6. Trichord placement is fixed, but internal content may be ordered freely. (In the first and third order sets, pitches of complementary order number must not create the interval 6:

0 2 1 3 5 4 9 10 11 8 6 7)

d ≠ 6

2) P∪P∪R∪R extends at t=3, when $Hb=Ha^{t}$ for any ordering of 'a'. Trichords are then ordered freely.

The next example illustrates an eight-part extension of the second order semi-combinatorial set, I34. (Example 26 may be so extended if P and I partitions are (2 1 1 2 1 2 2 1) and if R and RI partitions are (1 2 2 1 2 1 1 2).

Example 27

P	0	3	7		6	9	1		8	11	4	5	2		10		P	6	9	1															
I	11	8	4		5	2	10		3	0	7		9	6	1		I	5	2	10															
RI	1		6	9	7		0	3	10		2	5	4		8	11	RI	7		0	3														
R	10		5	2	4		11	8	1		9	6	7		3	0	R	4		11	8														
P	6	9	1		0	3	7		2	5	10		8	11	4		P	0	3	7		et													
I	5	2	10		11	8	4		9	6	1		3	0	7		I	11	8	4															
RI	7		0	3	1		6	9	4		8	11	10		2	5	RI	1		6	9														
R	4		11	8	10		5	2	7		3	0	1		9	6	R	10		5	2														

Each member of the sub-group of order 8 is represented!

Third order sets are always capable of six and twelve-part combinations if the intervals 4 and 8 are excluded from the even numbered dyads (order numbers 0 1, 2 3, 4 5, etc.) of the basic set.

Example 28a

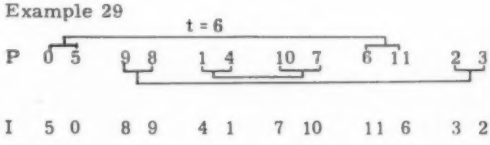
P	0	1	8	5	4	9	6	11	7	10	2	3																							
I	7	6	11	2	3	10	1	8	0	9	5	4																							
R	3	2	10	7	11	6	9	4	5	8	0	1																							
RI	4	5	9	0	8	1	10	3	2	11	6	7																							
P	8	9	4	1	0	5	2	7	3	6	10	11																							
I	11	10	3	6	7	2	5	0	4	1	9	8																							
	E		RI21		E		E		B		E																								

Example 28b

P	0	1	8	5	4	9	6	11	7	10	2	3																							
I	7	6	11	2	3	10	1	8	0	9	5	4																							
P	4	5	0	9	8	1	10	3	11	2	6	7																							
I	11	10	3	6	7	2	5	0	4	1	9	8																							
P	8	9	4	1	0	5	2	7	3	6	10	11																							
I	3	2	7	10	11	6	9	4	8	5	1	0																							
	E		E		E		E		E		E																								

Example 28b combines with itself at transposition 6 to produce a twelve-part statement which satisfies (6^2), (2^6), and (1^{12}). If the three dyads of the first hexachord represent the intervals 1(11), 3(9) and 5(7) at any even order position, and the complementary hexachord transposes these dyads into any even numbered order position at the tritone, P(xy)⟶I(yx) at the same order position.*20

Example 29



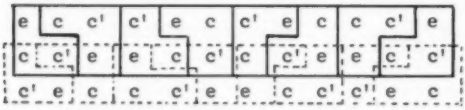
In addition, if the basic set is in the form aa^+/aa^+, and every element taken with the corresponding element of RI is an expression of the interval 2, 6, or 10, even greater combina-

torial flexibility (of which Example 30 is but an instance) is obtained.

Example 30

P	0	11	3	8	7	4	5	6	9	2	10	1
I	1	2	10	5	6	9	8	7	4	11	3	0
R	7	4	8	3	0	11	10	1	2	9	5	6
RI	6	9	5	10	1	2	3	0	11	4	8	7
P	4	3	7	0	11	8	9	10	1	6	2	5
I	9	10	6	1	2	5	4	3	0	7	11	8
R	11	8	0	7	4	3	2	5	6	1	9	10
RI	2	5	1	6	9	10	11	8	7	0	4	3
P	8	7	11	4	3	0	1	2	5	10	6	9
I	5	6	2	9	10	1	0	11	8	3	7	4
R	3	0	4	11	8	7	6	9	10	5	1	2
RI	10	1	9	2	5	6	7	4	3	8	0	11

Example 30 satisfies (6^2) for adjacent forms, (3^4) for each block of forms related by all four operations, and (1^{12}) for all forms employed. Furthermore, (2^6) is satisfied by P⌣ I⌣ P⌣ I⌣ P⌣ I or by the remaining six forms; resultant tetrachords are ordered such that eight part combinations exist if each set in block one is in the form (1 2 1 2 2 1 2 1) and each set in block two is in the form (2 1 2 1 1 2 1 2). The twelve part combination may be represented as a series of overlapping aggregates:



Thus Example 30 may be regarded as a construction derived by operation upon 2, 4, 6, or 8-part combinations. Note that the entire subgroup of order 24 is represented if both forward and backward readings are taken. (This is not possible with the set of Example 29).

The set's combinatorial potential is then a function of relations

within and between its parts. To the aforementioned equal part partitions (6^2), (4^3), (3^4), (2^6), (1^{12}), I now add the unequal two part partitions: (1 11), (2 10), (3 9), (4 8), (5 7).

The method by which combinatorial properties are uncovered remains unchanged. Define the set as follows: (3) (6 3) or (36) (3).

Then P∪P=A if content P (9-11) is a transposition of content P(0-2).

P∪I=A if content P (9-11) is inversionally similar to content P(0-2).

P∪RI=A if content P(9-11) is inversionally symmetric.

All combinations are possible if outer trichords are inversionally symmetric and related. (0 2 7 .....11 1 6).

Example 31

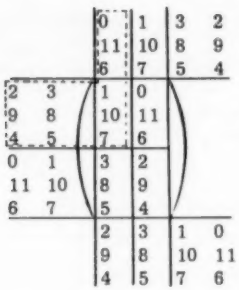
P							0	10	11	1	9	6	8	5	7	2	3	4		
P	8	6	7	9	5	2	4	1	3	10	11	0								
P							0	10	11	1	9	6	8	5	7	2	3	4		
I	2	4	3	1	5	8	6	9	7	0	11	10								
P							0	10	11	1	9	6	8	5	7	2	3	4		
RI	6	7	8	3	5	2	4	1	9	11	0	10								

Unequal part partitions may be applied to subsets; in the next example the partition (1 3) of the linear forms results in (3 9) for the derived harmonies, and the combination is extended accordingly:

Example 32

		0	1	3	2
		11	10	8	9
		6	7	5	4
2	3	1	0		
9	8	10	11		
4	5	7	6		

and extended once again by (2 1 1):



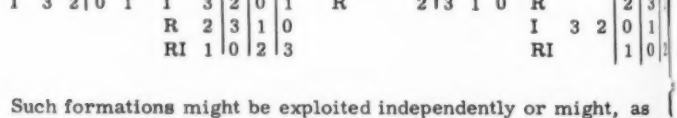
Of the 38 source pentads (see Table VII), nine, those which preserve content on inversion, are all-combinatorial. (set #1, 5, 7, 10, 17, 25, 32, 35, and 38) Seven sets combine with I or P (not both) and are termed I/P types; twenty-one are I type; one set, the inversionally symmetric set #16, has no special properties. The sign (=) is employed in Table VII to signify content preservation between P and I forms. The sign ($\neq$) indicates the lack of content preservation and refers to I/P types.

5/6/7=9/10/11 is read $(P_5=I_9)\neq(P_6=I_{10})\neq(P_7=I_{11})$.

Pentad-hexachord relations depend upon the placement of the remaining two notes: 0 1 2 3 4 $\binom{5\ \ 11}{11\ \ 5}$ 6 7 8 9 10 or 0 1 2 3 4 $\binom{5\ \ 6}{6\ \ 5}$ 7 8 9 10 11.

Heretofore aggregate formations of twelve notes have been formed at the part level. But any subset might be regarded as a "total set" whose parts would then be subject to aggregate formations equivalent in content to the subset. The result would be internal pitch repetitions that are operationally structured.

Example 33



Such formations might be exploited independently or might, as

table VII

SOURCE PENTADS

No.	Pentad	10 intervals 1 2 3 4 5 6	P tr.	I tr.
1	01334	4 3 2 1 0 0	5/6/7	• 9/10/11
2	01235	3 3 2 1 1 0	6	/ 8/9
3	01245	3 2 2 2 1 0	6	/ 11
4	02347	2 2 2 2 2 0	6	/ 1
5	01346	2 1 2 3 2 0	6	• 10
6	01458	2 0 2 4 2 0	2	/ 11
7	01359	2 1 2 3 2 0	6	• 8
8	02357	1 3 2 1 3 0	6	/ 11
9	01358	1 2 2 2 3 0	6	/ 7
10	02479	0 3 2 1 4 0	6/1	• 10/5
11	01236	3 2 2 1 1 1		10/11
12	01256	3 1 1 2 2 1		9
13	01237	3 2 1 1 2 1		10
14	01246	2 3 1 2 1 1		9/11
15	01348	2 2 3 1 1 1		11
16	01356	2 2 2 1 2 1		
17	02346	2 2 2 2 0 1	5/7	• 11/1
18	01247	2 2 2 1 2 1		10
19	01257	2 2 1 1 3 1		11
20	01347	2 1 3 2 1 1		9
21	01457	2 1 2 2 2 1		3
22	01248	2 2 1 3 1 1		7/11
23	01258	2 1 2 2 2 1		11
24	01378	2 1 1 2 3 1		5
25	01478	2 0 2 3 2 1	2/10	• 10/6
26	01257	1 3 1 2 2 1		9/11
27	01368	1 2 2 1 3 1		5/10
28	01468	1 2 1 3 2 1		3/11
29	02358	1 2 3 1 2 1		9
30	02458	1 2 2 3 1 1		3/11
31	01469	1 1 3 2 2 1		11
32	02469	0 3 2 2 2 1	1/11	• 7/5
33	01267	3 1 0 1 3 2	9	/ 11
34	01367	2 1 2 1 2 2		11
35	01268	2 2 0 2 2 2	3/9	• 9/11
36	02368	1 2 2 2 1 2		1
37	01369	1 1 4 1 1 2		11
38	02468	0 4 0 4 0 2	1/3/5/7/9/11	• 9/11/1/3/5/7

in the next example, be conjoined with twelve note aggregate formations.

Example 34

P	0	1	3	2	10	11	4	5	6	9	8	7
	3	2	0	1					8	7	6	9
RI	6	5	4	7	8	9	2	3	11	10	0	1
	4	7	6	5					0	1	11	10
P	8	9	11	10	6	7	0	1	2	5	4	3
	11	10	8	9					4	3	2	5

A unique world of linear-vertical associations, itself a segment of the "theoretical" universe, embracing all possible set orderings, of such relations, is therefore created by the initial selection of a basic set. For the set defines the content of its parts and the order of their occurrence, and these in turn determine the classes of combination available, and thus the levels within which, and the methods by which, aggregates may arise through the operation of combination. Further partitioning of the set parts together with the selection of a specific combination results in "structured aggregate-formations".

That the subject of set and aggregate partitions is, at the close of this paper, far from exhausted, may be seen from a single abstraction. If we assume the set partition (6^2), the resultant aggregate-forming combination $\begin{matrix} 6 & 6 \\ 6 & 6 \end{matrix}$, and an initial aggregate-formation $\begin{matrix} 5 & 1 \\ 2 & 4 \end{matrix}$, then our next choice must be made from among the following possibilities:

1 5, 2 4, 3 3, 2 4, 4 2, 1 5, 2 2 2, 1 2 3, etc., etc., ...
5 1 4 2 3 3 1 5 1 5 3 3 2 2 2 3 1 2

Attending such decisions are the foreground phrase members, their harmonic associates, and, more significantly, the further linear and vertical extensions of 'S'.

5 1 2 4
2 4 5 1
4 2 1 5
1 5 4 2

references

1 A complete discussion of this idea will be presented by the author in a future article.

2 Complements are not included in the table.

3 Remember that the interval 1 stands for itself as well as its complement 11. Thus intervals are summarized by the digits 1 .... 6.

4 For example, in source set 012457 the intervals 1, 2, 3, and 5 occur three times each; the interval 4 occurs twice and 6 occurs once. The total is 15.

012457
1346
235
13
2

5 Glossary of symbols:

P, I, R, RI stand for the four forms: prime, inversion, retrograde, and retrograde-inversion.

T	means	"is a transposition of"
t		"transposed"
*		"inverted," "the inversion of"
≠		"not equal to"
⌣		"combines with," "combined with"
d		"interval"
i		"order"
j		"pitch"
A		"aggregate"
⟶		"maps into," "becomes"

6 The equivalent statements from Babbitt's article in the April 1961 issue of this journal are here reprinted: $H_0 + T^tH_0 = A$
$H_0 + T^tIH_0 = A$
$H_0 + T^tRIH_0 = A$

In future references to combinations or classes of combination the transposer (t), or (T), as well as the content symbols (a), (b) will be understood (i.e., P⌣P, or P⌣P = A is taken to mean $Pa \smile Pa^t = A$ and $Pb \smile Pb^t = A$). When transposition numbers are absolutely necessary T_6P will be abbreviated P_6. P(0-3) indicates prime order numbers 0 through 3.

7 Such is the case in Webern's Piano Variations, Op. 27/I, where P⌣R, RI⌣I, creating fixed units of hexachordal harmony $x, y, y, x, x^+, y^+, y^+, x^+$ (type R) distinct from the type "A" linear hexachords.

8 The term "derivation" in its present context is not to be confused with the same term applied to the construction of the set itself.

9 I shall confine myself to the discussion of aggregate forming combinations although similar results are obtained in "non-combinatorial" combinations (see Webern's Kantate, Op. 31/V where R⌣R⌣R⌣R ≠ A, or his Op. 27/II where P⌣I ≠ A).

10 "Class of combination" signifies all combinations derived by operation upon a given combination (i.e., P⌣P: I(P⌣P) = I⌣I, R(P⌣P) = R⌣R etc.).

references

11 The transposition numbers for the preservation of hexachordal content are always even. Thus regardless of the combination employed the ratio of intervals odd to even is maintained within the subgroup since any number, x, plus an even number equals a number whose quality — odd or even — is the same as that of x.

12 See The Score and I.M.A. Magazine, June 1955, p.55.

13 If trichords are inversionally symmetric there will be a choice of "followings".

14 There are 43 tetrachords of which 15 are inversionally symmetric ($a = a^{+t}$) and 14 are in the form 'a' to 14 in the form 'a$^+$' where 'a' $\neq$ 'a^{+t}'. Defined by their prime forms the number is reduced to 29 (15 + 28/2) source tetrachords.

15

's'	0 1 3 6	0 6 1 3	1 3 0 6	'a'	0 1 2 3	0 1 3 2	'b'	0 2 3 5	0 2 5 3
	0 1 6 3	0 6 3 1	1 6 0 3		0 2 1 3	0 2 3 1		0 3 2 5	0 3 5 2
	0 3 1 6	1 0 3 6	3 0 1 6		1 0 3 2	0 3 1 2		2 0 5 3	0 5 2 3
	0 3 6 1	1 0 6 3	3 1 0 6		1 3 0 2	0 3 2 1		2 5 0 3	0 5 3 2

'c'	0 2 5 7	0 2 7 5	'd'	0 5 6 7	'e'	0 1 6 7	'f'	0 3 6 9
	0 5 2 7	0 5 7 2		0 5 7 6		0 1 7 6		0 3 9 6
	2 0 7 5	0 7 2 5		0 6 5 7		0 6 1 7		0 6 3 9
	2 7 0 5	0 7 5 2		5 0 6 7		0 6 7 1		
				5 0 7 6		0 7 1 6		
				5 7 0 6		0 7 6 1		

16 The 16 combinations of 0253 are listed in full:

P∪I (14, 37) 2	P∪R (15) 4	P∪I∪R (13, 16, 16, 25)		
I 9 7 4 6	R 7 9 6 4	I 9 7 4 6	R 7 9 6 4	Type I9
P 8 10 1 11	P 8 10 1 11	RI 10 8 11 1	RI 10 8 11 1	
P 0 2 5 3	P 0 2 5 3	P 0 2 5 3	P 0 2 5 3	
I 1 11 8 10	R 11 1 10 8	I 1 11 8 10	R 11 1 10 8	(T_4) Type I9
P 4 6 9 7	P 4 6 9 7	RI 6 4 7 9	RI 6 4 7 9	
P 0 2 5 3	P 0 2 5 3	P 0 2 5 3	P 0 2 5 3	
I 1 11 8 10	R 11 1 10 8	R 11 1 10 8	I 1 11 8 10	(T_8) Type I9
I 9 7 4 6	R 7 9 6 4	I 9 7 4 6	R 7 9 6 4	
P 0 2 5 3	P 0 2 5 3	P 0 2 5 3	P 0 2 5 3	

P∪P (48)4	P∪RI (26)4			
P 8 10 1 11	RI 10 8 11 1	RI 10 8 11 1	P 8 10 1 11	Type F
P 4 6 9 7	RI 6 4 7 9	P 4 6 9 7	RI 6 4 7 9	
P 0 2 5 3	P 0 2 5 3	P 0 2 5 3	P 0 2 5 3	

NB! All possible vertical pitch relations are obtained:

	4	6	7	9	4	6	7	9	1	8	10	11
	1	1	1	1	8	8	8	8	4	4	4	4
x =	0	0	0	0	0	0	0	0	0	0	0	0

1	8	10	1	8
4	4	4	7	7
0	0	0	0	for all values of x.

17 Two-part oblique combinations made possible by unequal binary partitions such as (4 8) will be discussed at the close of this paper.

references

18 Statements such as aa^+/aa^+, or ab/ab, etc. always assume the transposer (t).

19 Results are summarized in Table I.

20 Note that this property is available whenever Ha = Hb^t8.

Type A set P 0 5 1 4 2 3 11 6 7 10 9 8

I 5 0 4 1 3 2 6 11 10 7 8 9

THEORY FORUM

R. WILDING-WHITE

There are probably few musical terms that have been surrounded by as much controversy and ambiguity as the term tonality. Readers of this journal are aware of the amount of theoretical study it still incites, and, far from being limited to the tonal music of the eighteenth and nineteenth centuries, the term is applied to a musical spectrum that ranges from anthro-musicology to serial composition.

It would appear to me that the basic problem is a semantic one. The word tonality has been assumed to denote one concept, whereas in fact it has been used to denote two quite different concepts. The best way to illustrate this duality is to quote one or two excerpts from the lengthy list of definitions of the term.

[Tonality is] the art of combining tones in such succession and such harmonies or succession of harmonies, that the relations of all events to a fundamental tone is made possible. (M. Armitage (ed.), Schoenberg (New York, 1937) 47)

Tonality in this case is defined as a matter of relationships between simultaneous or successive pitches; it is a dynamic concept implying such an equation as (pitch x) = (f) (pitch y), and suggesting such terms as tonic or root.

I say, then, that tonality resides in the melodic and harmonic affinities of the sounds of the scale, which determine the successions and aggregations of these sounds. The composition of chords, the circumstances which bring about their modification, and the laws of their succession, are a necessary result of this tonality. Change the order of the sounds of the scale, distribute their intervals differently, and the majority of the harmonic relationships cease to exist. (F.J. Fétis, Traité..... trans. in M. Shirlaw, The Theory of Harmony (London,1919) 326)

Here the originator of the term is using it in order to give a qualitative description. The term defines the characteristic of a group of pitches, it implies no function; rather, it defines a group of functions and suggests such a term as keys.

Returning to the first quotation, we should note that Schoenberg does not say that the relationships are due to natural law but rather that they are a matter of art. This is contrary to the classical concept, which considers the harmonic series as the basic law of music. By this law explanations have been found for the grouping of chords in families of inversions, the reason for the sense of key in a melody because of its having an inherently implied harmony, and last, but not least, the relationships of tonality.

Unfortunately there is not space enough to enter into a detailed discussion of the so-called acoustical theory. Suffice it to say that it has led to such an endless series of contradictions, both in its classic and more recent applications, that it would not be wise to regard it as natural law. (For example, Hindemith's Craft of Musical Composition. In some of his writings Schoenberg also supports this theory.) One should look elsewhere for an explanation of the phenomena of tonality, inversion, and so on.

The best way to begin is by an analogy with the mathematical idea of sets. Since in dealing with music we are dealing with

human limitations, and cannot therefore speak in terms of infinity, we are only working by analogy.

Without going into a strict definition, a set can be described as an arbitrary system of relationships. For example, the following systems can be called sets: all numbers, all whole numbers, all odd numbers. In these three examples it is apparent that (for any value less than infinity) the first of these contains the next two and the second contains the last; and that any number of individual relationships can be set up even within the smallest of the three. The same reasoning applies to the following examples: all pitches, all pitches a semitone apart, all pitches described by the C major scale (assume tempered semitones).

The sets which describe music can be classified into three kinds: gamut, scale, and formula.

The gamut of a musical system is the largest permissible set. The gamut of all music is naturally the total of all audible pitches; that of contemporary western music is a 12-note equally tempered scale cyclical at the octave.

A scale is a set contained within the gamut; the largest possible scale is identical with the gamut. In western music the largest possible scale is the 12-note tempered scale, and is identical with the gamut. The diatonic scales are smaller.

Finally, a formula is an individual relationship (a limited set) contained within the scale.

It is by no means necessary that any of these three be cyclical at the octave, even though gamuts and scales generally are. Be it noted, however, that if the gamut is cyclical at the octave, it can contain scales which are or are not cyclical at the octave; but if the gamut is noncyclical at the octave, all scales must be so too.

These three types of sets are arbitrary systems. It is more than possible that they are the result of long-term musical evolution; and, in fact, it is suggested quite plausibly that musical evolution is a social process that leads from formulas to scales to gamuts.

The process can be described as follows. If we imagine a musical system consisting of only one formula — say a chant on CEF — this formula will be scale and gamut as well. With the

appearance of a second formula — say EFA — the two formulas are identical with two scales contained in, but no longer identical with, the gamut which is now CEFA. These two scales can develop further formulas which are not identical with the scales, such as CE, EF, EA, etc. Furthermore, within the new gamut hybridization may take place, creating new formulas, such as CA, or new scales, such as CEA or CFA. As this takes place the original formulas lose their identity, and, if the process is uniform, the musical system evolves a single scale (CEFA) which is identical with the gamut. This is identical with the process termed blending, in modern theories of linguistics. (The interested reader is referred to an excellent article by C.F. Hochet, The Origin of Speech, in Scientific American, CCIII (1960) 89-96.)

Transposition of individual formulas within this gamut is selective. The third, CE, can be transposed to the pitch F, but not to the pitch E, which requires the introduction of the new pitch G#. If this new pitch is introduced, a new gamut develops (CEFG#A) which is once again a set larger than any of its component scales or formulas. Musical evolution, of course, cannot be expressed in such pat terms, since it proceeds by a series of conflicts and compromises; but it is interesting to note that Sachs believes that music is on the road to standardization, and that all musical systems eventually develop equal temperament. (C. Sachs, The Rise of Music in the Ancient World, (New York, 1943) 283)

The preceding material introduces, however, an important problem: what is it that defines a musical set? It would appear that this is a purely relative matter and that a musical set is an arbitrary grouping, possibly even where it is the result of a cultural evolution.

Take, for example, two sets: (A) containing pitches 1, 2, and 3, and (B) containing pitches 4, 5, and 6:

Case 1: 123231 455654

A B

Case 2: 123231 -- 634251 ----654546

A A+B B

Case 3: 12323163215133431

+ + +

A piece of music will be cast in a total set comprising all of the pitches used; in the above cases, A+B. This total set can be broken down into smaller sets, or subsets, which, however, can only be considered such for as long as they maintain their identity. In Case 1 the subsets are clearly defined; in the central section of Case 2 both subsets lose their identity in favor of the total A+B; in Case 3, set A is clearly dominating, and it is doubtful if pitches 4, 5, and 6 can be reasonably described as a subset.

The following extract from Dr. Faustus by Thomas Mann (a free paraphrase of Schoenberg's Harmony) will serve as an illustration:

> To illustrate the meaning of the word [ambiguity] he played me chord-progressions belonging to no definite key; demonstrated for me how such a progression fluctuates between C major and G major, if one leaves out F, that in G major turns into F-sharp; how it keeps the ear uncertain as to whether that progression is to be understood as belonging to C major or to F major if one avoids the B, which in F major is flattened to B flat. (New York, 1948, p.47)

Let us see how this applies to the term tonality. As I pointed out earlier, two separate concepts have been described by the one term. Traditional theory has used the word to apply to a group of notes (as in a scale or key) and to apply to a single note (as in tonic). One further quotation will help to make this ambiguity clear:

> . . . [tonality] coincides to a certain extent with . . . key, in so far as it refers not merely to the relation of tones with one another but much more the particular way in which all tones relate to a fundamental, especially the fundamental of the scale whereby tonality is always comprehended in the sense of a particular scale. Thus, for example, we speak of a C major tonality. (Armitage (ed.), op.cit., p.1,268)

A separation of these two ideas will clarify the confusion, for in one case we have a characteristic of a certain system of pitches, and in the other an artificially created system of relationships. Tonality is a function of a pitch set, be it expressed as a gamut, scale, melody, chord, or any other arrangement. It is, in fact, identical with the set in that it is simply the sum total of all the possible combinations of the members of that set. (Since in a musical texture variations in duration, tone color, etc., are involved, every set has an infinite number of variations if the factors other than pitch are considered.)

Returning to a numerical analogy, the set of all even numbers expresses no preference for any member, but the set can be expressed as a function of one or more of its members (for example, 2, alternate pairs, all multiples of 8).

Thus tonicality (I have borrowed Reti's term) represents a procedure: it is a measure of bias and represents the relative importance given to each of the subsets contained in a given set.

It is important to note that this is not just the hierarchy of individual pitches, but is rather a hierarchy that can include groups of pitches. It has many times been suggested that the root of the key of C major as it is used in a traditional context may not be the note C, but the C major triad. (As Joseph Schillinger put it in one of his rare moments of lucidity, the music of the tonal period represents the hegemony of the C_1 chord — Schillingerese for root triad.)

In order to investigate this theory on a more practical level, I resorted to the study of the nature of the scales contained in the contemporary western gamut. Tonality describes the characteristics of a pitch set, and it makes no difference whether these pitches are presented as a melody, a chord, or a scale. However, a scale arranges the pitches in consecutive order, and thus presents them in the easiest form for purposes of study. A symbolic system was designed to describe each scale in terms of its interval configuration. This can best be explained by an example:

C(12) - 2(2)1(1)3(2)1(1)-RC-M1

The first figure, C(12), indicates that it is cyclical at the octave. The next series of numbers describes the configuration of the scale; numbers not in brackets give the number of successive identical intervals, the numbers in brackets indicate the type of interval, in semitones. The example above describes the major diatonic scale. The following two groups of symbols do not describe a feature of the scale, but specific versions of it: RC reads "root C" and specifies which transposition is being used; M1 reads "mode 1" and indicates which note is referred to as tonic.

By describing the intervals in terms of semitones, this notational system is limited to the tempered chromatic scale. It could, however, be extended to cover any musical system by redefining the interval in terms of commas, for instance.

The term root in this case is not used in its traditional sense, but simply indicates the pitch from which the configuration is spelled out in ascending order. Thus the scale above can have 12 roots (C, C#, D, ...) which correspond to the 12 transpositions of the diatonic scale. The term mode is used in the same sense as in church modes, that is, to indicate which note of the scale is to be considered as tonic.

By this system of classification, the transpositions and modes of a scale are not considered as independent scales, but are grouped as one family and considered as one scale. Thus the tabulation of all possible scales within an arbitrary system becomes possible in a systematic manner.

As a matter of convenience, I limited the tabulation of possible scales by the following arbitrary conventions: (1) The 12-note tempered chromatic gamut was assumed; (2) All scales are cyclical at the octave (C(12) scales); (3) The largest interval included was the major third (4 semitone units). The total number of scales for this situation is 232.

Each of these scale families is not necessarily in one tonality. The example given above is the C major scale spelled out from the note C. The following version,

C(12) - 2(2)1(1)3(2)1(1)-RG-M4

is the fourth mode of the G major scale, and thus differs from the first version by one note, F#, which has replaced F natural. Thus a change of tonality has taken place even though the tonicality is constant.

Like the majority of the scales, the members of the diatonic scale family are asymmetrical in their configuration. There are, however, 14 scales that are symmetrical. This means that they are not only cyclical at the octave, but also at some smaller interval. These fourteen scales are:

1	C(1) - 1(1)			Chromatic scale
2	C(6) - 4(1)			
3	C(4) - 2(1)	1(2)		
4	C(6) - 2(1)	2(2)		
5	C(3) - 1(1)	1(2)		Bartok usage
6	C(2) - 1(2)			Whole tone scale
7	C(3) - 1(3)			Diminished seventh chord
8	C(4) - 1(4)			Augmented triad
9	C(6) - 3(1)	1(3)		
10	C(4) - 1(1)	1(3)		Vaughan Williams usage

11 C(6) - 2(1) 1(4)
12 C(6) - 1(2) 1(4)
13 C(6) - 1(1) 1(2) 1(3)
14 C(6) - 1(1) 1(3) 1(2) (Mirror of no. 13.)

These scales can have only a limited number of transpositions, the rule being that the number of transpositions is equal to the number of semitone units between the points of symmetry. Thus, all asymmetrical scales, cyclical at the octave, have 12 transpositions; if they are cyclical at the minor third, they have three, and so on. The first figure in the code, then, also indicates the number of transpositions.

Any scale can acquire tonicality by making any one of its pitches act as tonic; and, similarly, any of the transpositions of the scale can be given the same choice of tonics. In the case of asymmetrical scales, the number of modes thus made possible is equal to the number of pitches of the scale. In the case of symmetrical scales, however, the number of modes is limited and is equal to the number of items in the second unit of the code within the points of repetition. In the five examples given above, the first three have only one mode, and the other two have two modes apiece.

Unlike transposition, a change of mode does not involve a change of tonality. Thus, a mode is a delicate creature and maintains its identity only so long as the tonicality that defines it is maintained, a fact proved again and again in musical history.

Three possible ways exist for altering the tonal arrangements possible within the 12-note chromatic gamut:

Mutation. (I use this word for lack of any precedent.) The configuration of the scale itself can be altered, e.g. the major diatonic scale altered to the whole-tone scale:

C(12) - 2(2)1(1)3(2)1(1) - C(2)-1(2)

Transposition (or modulation). The scale can be spelled from any note in the gamut. Thus, C diatonic becomes G diatonic:

C(12)-2(2)1(1)3(2)1(1)-RC.

Change of mode. By changing the note defined as tonic, any given scale can change in mode. Thus, C Ionian to C Dorian:

C(12) -2(2)1(1)3(2)1(1)-RC-M1 - Idem, M2.

Any two or all three of these operations can occur simultaneously. (1) By transposing and changing the mode the same tonic can be maintained (Ionian of C can become the Myxolydian of F).

C(12)-2(2)1(1)3(2)1(1)-RC-M1→ Idem, RF-M5.

(2) By mutation one scale can be absorbed into another: the natural pentatonic (F# G# A# C# D#) can, by the addition of B and E#, become the diatonic F# major, or by the addition of E and B# become the Phrygian of C# major

C(12)-2(2)1(3)1(2)1(3) - C(12)-2(2)1(1)3(2)1(1) - RF#-M1→
C(12)-2(2)1(3)1(2)1(3) - C(12)-2(2)1(1)3(2)1(1) - RC#-M4.

(3) It is possible to accomplish all three functions at the same time without changing the tonic. C major can become the fourth mode of F pentatonic.

C(12)-2(2)1(1)3(2)1(1)-RC-M1→C(12)-2(2)1(3)1(2)1(3)-RF-M4.

This discussion of the various possible scale configurations does more than provide a methodology for the study of pitch sets: it helps to explain several important musical problems.

The major triad is a scale whose code by the above system is C(12)-1(4)1(3)1(5). It is a nonsymmetrical scale and thus has twelve transpositions and their corresponding three-pitch tonalities within the western gamut; it can be given three tonics, that is, is capable of three modes. These three modes are what have been called its inversions. As I have said, modes are delicate, and easily lose their identity. This is due to the large number of identical formulas that they contain, as discussed. Just as the diatonic scale is a subset of the gamut, so the triad is a subset of the diatonic scale (and other scales, obviously), and the individual formulas (such as EG and its variants in time, dynamics, etc.) are subtonalities of the triad. The particular grouping of formulas found in the scale described above is the triad and its inversion. The reasoning applies to any chord.

Symmetrical scales are subject to such loss of identity that it becomes a special feature of their type. For example, the scale C(2)-1(2), the whole tone scale, is symmetrical, allows only one transposition, and has only one mode. In other words, complete duplication of formulas exists in all modes and in all but the one transposition of this scale. Similar situations exist for the symmetrical scales which were discussed above; thus, it can be seen why chords like the augmented triad, the diminished seventh, or (to include one not cyclical at the

octave) the chord in successive fourths, are referred to as indeterminate chords. The reductio ad absurdum is the scale C(0)-1(1), the unison. Similarly, the chromatic gamut, C(1)1(1), is an indeterminate. In a larger framework this applies to cyclicality at the octave, and explains the usually unquestioned belief that a melody and its octave transposition are identical. Actually they are not; they only appear to be so.

The use of the indeterminate chords by composers in order to introduce ambiguity wilfully is a standard technique. Also, note that traditional harmony relies heavily on the idea of pivot chords, common tones, etc., and it is contradictory that so many theorists should reject the idea of modulation expressed in Dr. Faustus, quoted above.

Indeterminacy in a pitch set does not mean that the set cannot be given a tonic, it only means that the formulas in the set will not determine the tonicality; this tonicality will have to be imposed on it by some such technique as the use of pedal point (see Voiles in Debussy's Preludes, Book 1).

Nonmember tones constitute a problem in the theory of every biased musical system — be they called imperfections, dissonances, nonchord tones, or what have you. Thus, in the West today, all audible pitches are biased in favor of the chromatic gamut. In traditional practice, this gamut is biased in favor of the diatonic scales, which are in turn biased in favor of the triads they contain. Thus, to the spectrum of possible tonalities a system of tonicalities is applied.

Similarly, within a consistent system, whenever a change of tonality is permitted (by mutation, modulation or change of mode), an area will exist at the junction point which, though determinate all by itself, is indeterminate in relation to the previous and future sections of the composition. Thus nonmember tones will exist for two reasons: (1) by the transient expansion of a tonality, that is, by the transient addition of pitches outside of the permitted tonality; and (2) by the change of tonality (or tonicality). There is a striking similarity between the nature of the nonmember tone and the general definition of noise as "that part of the signal which is not a part of the message".

An obvious conclusion from this would be that the measure of the redundancy of various pitches would give us a measure of the laws of tonicality applicable to a particular piece (a form of Markoff process, in other words). I would doubt, however,

that this would give us an accurate picture; a composer or listener familiar through past experience with the laws of a given musical system would hear the music differently from the implications of its mechanical analysis. (He would be essentially subjectively altering the redundancy of the work. On a bad day when the pitchers bean all the batters, a Martian would go away with an interesting but inaccurate impression of a game of baseball).

Strictly speaking, there is no such thing as a non-member tone, there is only a change of tonality and tonicality; but subjectively, there are non-member tones and a value judgment is required before the system of tonicality of a given work can be determined.

By definition, it is impossible for a work to be atonal, but it is not impossible for it to be atonic. The only way to achieve complete atonicality, however, is to ensure that all of the pitches of a set are evenly distributed throughout a musical texture; any form of bias (in rhythm, dynamics, etc.) will introduce to one degree or another a tonicality. In some serial compositions this situation is almost achieved. Evenly distributed, by the way, is not the same as randomly distributed. I have myself applied the serial techniques using both chance sequences and random sequences in writing works, and as one would expect, tonicality arises, though in a chance or random manner. It is interesting that Schoenberg, Leibowitz, Berg, and others often used the term monotonality to describe the results of serial composition.

At the other end of the line, it is equally foolish to say that a work must have one and only one tonic, or that this tonic has to be a single pitch, or that all of the various other tonalities are in some way attracted or repelled from this tonic. Tonicality can be absent (it rarely is) in a completely pantonal situation, but if it is present, it can be a function of one or of a group of pitches. There is also no reason to believe that a work has only one tonic, or to attempt to create theories which will force a musical work into this idea.

When one speaks of the tonicality of a work, one is speaking either of the average or of the predominant tonality of the work. If this predominant tonality exists, other tonalities can be compared against it; thus we can speak of tonal relationships. These are of two kinds:

Qualitative. Tonalities with the most pitches in common ob-

viously hybridize the most easily, have the least contrast, and can therefore be loosely referred to as closely related; those with the least pitches in common are the opposite.

Quantitative. The relative importance of a tonality with a musical work is in great part a function of its frequency of appearance. This, however, can be affected by the particular context involved, reinforcement by rhythmic texture, and all of the other techniques that a composer uses to create variation of emphasis. It is not enough to subject a work to a harmonic analysis or to a statistical scrutiny (although the latter is probably more accurate) in order to establish its tonicality.

I cannot give here a simple method for establishing the tonicality of a work, since far more study of this aspect alone of the problem is needed. It is my surmise that if the redundancy of a work were graphically represented in a progressive form (not its total or average), a clear picture of a musical work as a form of communication would be achieved; however, this redundancy would have to be expressed not only in terms of tonalities, but also in terms of melodic formulas, of meter and of rhythmic formulas, of tone color, dynamic level, and (in the case of nonmonaural music) of direction.

* * *

FORUM

NORMAN CAZDEN

In a previous communication (this journal, V (1961) 113-128) we have examined musical staff notation mainly from the general standpoint of communications theory. Our discussion was focussed on those features of staff notation which exhibit its origin in and its dependence upon the practical mnemonic needs of vocal and instrumental performance and learning, and especially on those features which emphasize its indelible links with the musical system it was called upon to serve. Thereby it was hoped that certain characteristics of that musical system, necessarily reflected in its notation principles, would emerge more clearly defined. These characteristics, judged abstractly, would otherwise appear as distortions, as departures from expected uniformities, as flaws; they have sometimes been criticized thus, thoughtlessly, as witness the

difficulties that adherents of the twelve-tone cult have met with over simple names for intervals, and a few other matters.

Staff notation reflects in its historically developed forms a strictly musical order of logic, and more precisely an order of logic pertaining peculiarly to the diatonic system of recent Western tonality. Within the framework of this systemic musical service alone, the current forms of staff notation, despite inadequacies of detail, are useful and rational, while from the standpoint of abstract or verbal orders of logic or of other possible musical systems these forms are in many respects hopelessly irrational.

The outcome of our analysis was the proposition that musical staff notation can lend itself to non-musical use as a communications code, since it is sufficiently rational to permit formulation in its terms of anything that may be stated in words. At the same time, the abstractly irrational properties of staff notation ensure that such a notation code could be so constituted that its mode of operation would be immune to deduction by anyone not apprized of its encoding method. The result is, on the one hand, that elusive ideal of communications codes deserving the term perfect cipher, and on the other hand a novel, fluent and fruitful technique for the composition of non-music, a diversion much indulged in of late though still hampered by inferior methods.

In order to test the validity of this proposition, two verbal messages were presented in notation cipher form. Example 1 in the cited communication (p. 123-124) presented a non-music composition for which neither the cipher key nor the translation was provided. Were this example to have proven susceptible to decoding, the demonstration would have failed at once. Otherwise Example 1 was to serve as proof for those to follow. Example 2 (p. 126-127) was similarly a non-musical message conveyed through the same notation code, still without the key to the cipher but now with a translation reference provided (the opening of Lincoln's Gettysburg Address). It was maintained that with its translation known, if the encoding principle could not be deduced from Example 2 and effectively applied to Example 1 as a check, then the cipher must be admitted to be inherently unsolvable.

The stated demonstration was presented as a challenge to the reader, with the promise that the solution would be forthcoming, together with due acknowledgment of any successful solutions for Example 1 that might have been received in the

interim. Despite an exceptional relaxation of the usual contest rules governing the enclosure of box tops or the ineligibility of staff members and their near relatives, it must be reported that up to the time of this writing no such successful solution has reached me, and no solution proposed has even approached the cipher principles involved. To this extent we may regard the original proposition as validated.

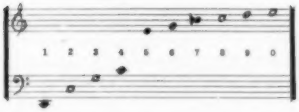
But in order to show that the concept of a notation code is more than the latest of sensational pseudo-theories now in vogue for the justification of diverse styles of non-music, I append here the promised Example 3, which is a cipher message with both its translation and its encoding or decoding instructions made plain, along with the solutions for Examples 1 and 2 which those instructions make possible.

Let us proceed by explaining the cipher employed, and then using it to encode a message. It should be recalled from my previous description that many such ciphers are possible; just how many has not yet been ascertained. Most of them will probably share the first principle of the present cipher, which is the arbitrary division of staff notation symbols into two large classes, those that shall convey the cipher message and those that shall conceal it. The multitude and variety of irrelevant and superfluous, yet normal, notation elements of this second class, are sufficient to frustrate solution of the cipher; neither their identity nor the need for their prior exclusion can be deduced, whether by rational means or by computers. Musical knowledge and experience are but further hindrances, as indeed they are for most dealings with non-music.

For our particular cipher, only notes proper shall convey the encoded message, and among these only certain half notes and eighth notes. But other note values, rests, accents, note stems and beams, slurs and a host of other common notation symbols shall be improvised freely, contributing to a normal appearance while serving as deliberate blinds. Since this is to be a relatively simple cipher form, as promised, the significant notes are to be read in their familiar way and in sequence; that is, there shall be no unwonted manipulations or puzzles involving clefs, inversions, retrogrades or other medieval formulas. Piano music format is the most convenient, though by no means the sole possible seeming medium of performance. Obviously, as with other types of non-music, no actual sounding performance is contemplated, though the results are of course less dreadful and unimaginative in sound

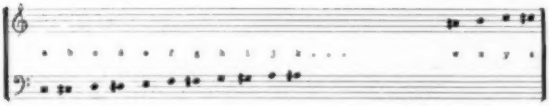
than those of the non-music styles now in the ascendance.

Confining our attention now only to the arbitrarily restricted class of notation symbols to be granted cipher values, certain half notes shall be assigned the values of numerals, and certain eighth notes the values of language letters. The numerals are to apply to their usual positions in the familiar representation of harmonic partials of the fundamental C, ignoring discrepancies of intonation. Since there is no zero partial to express the numeral 0, the tenth partial is used, in the manner of ace over king.

Example a

Any half note that falls upon one of these designated positions shall be read as the corresponding numeral. If more than one such half note is present at a time, then only the lowest shall be taken as significant. All others, along with half notes not corresponding in staff position to those harmonic series nodes, shall be meaningless.

Eighth notes shall be assigned respective letter values alphabetically, beginning with the note c to represent the letter a, and continuing chromatically to the note c#''' to represent the letter z.

Example b



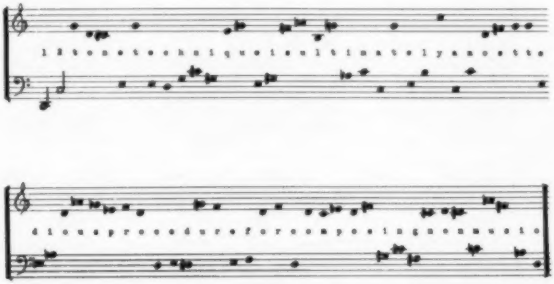
Enharmonic equivalents are to be treated as identities, so that the letter n, for example, shall be represented by either c#' or $d^{b'}$. Where more than one eighth note appears at a time, only the lowest shall be treated as significant. All others, and all eighth notes beyond the range specified, shall be meaningless. It may be observed how the lavish use of meaningless terms closely resembling those to be translated (modeled on

the promotional literature of non-music, which often resembles closely proper critical discussion of the art of music) here offers increased scope to the imagination of the sensitive composer, while simultaneously it helps provide camouflage for the cipher. Furthermore, we thus avoid infringing on twelve-tone patents, for without special treatments not used in this particular code, language letters do not retain intelligible status when squashed together and superimposed in heaps; whereas that is possible for serial terms without sacrificing their non-musical values.

We are now ready to encode a message that will constitute our Example 3. Let us choose as message the sentence: Twelve-tone technique is ultimately a most tedious procedure for composing non-music. (Lest some readers protest an unintended bias here, let it be declared that we would just as soon substitute any of such rival brand-names as 43-tone monophony, prepared-piano monotony, tone-clusters, ink-splatters, mystic modes, micro-tones, electronic doodles, pantonic or catatonic noodles, or tape-estries, as subject of the test sentence. We are not concerned here with the self-evident truth of the statement, but with the procedure for translating it into our notation code.)

First we set down in sequence the specified code representations for the numerals and letters required.

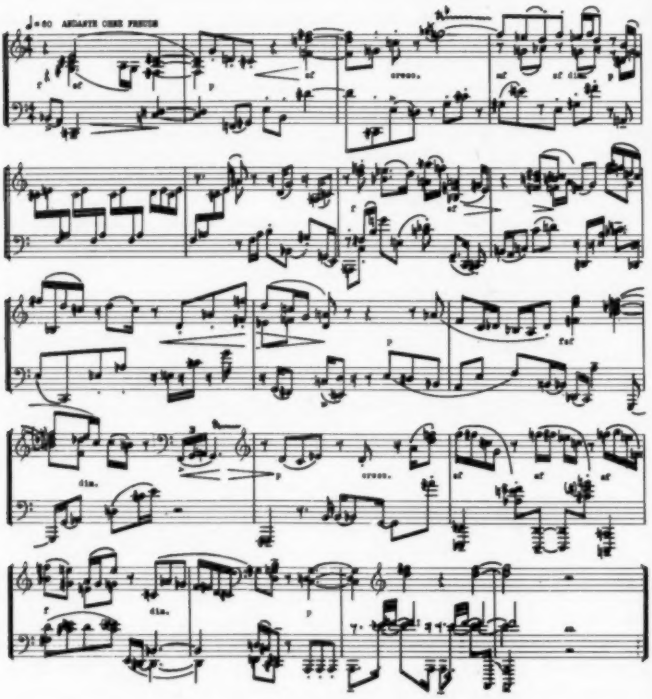
Example c



Next we space these coded forms at arbitrary time intervals, and improvise our non-music composition about them, as if embellishing a sacred cantus, hexachord or tone-row. In doing this we make sure to include the full paraphernalia of normal notational forms, for non-music is best presented in the outward garb of music. Care must be exercised merely

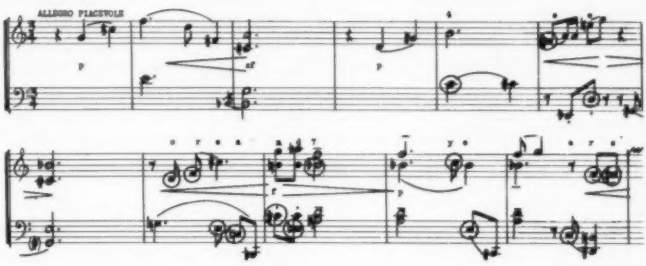
to avoid accidental insertion of true code terms not in the model. Potential hazards such as premature return to certain notes, which would offend twelve-tone sensibilities, or a noticeable concurrence with the letter frequencies established by professional decoding methods, are readily avoided by a judiciously random scattering of the significant data (a practice already familiar to theorists of non-music) and their saturation with exceedingly similar yet inconsequential fillers.

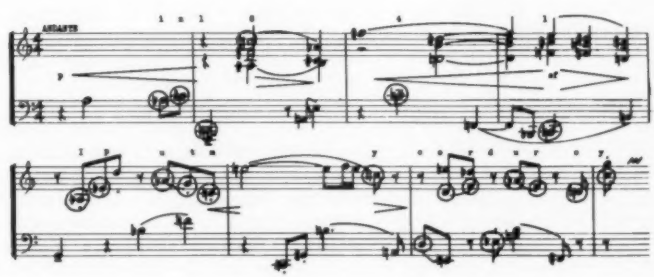
Example 3 as presented here is therefore but one instance of how the assigned message may be encoded.

Example 3

Innumerable variations on the theme are possible, retaining

the same cipher and the same message. They may be undertaken by any reader upon payment of the usual permission fee for thematics (New and slightly-used tone-rows also available at a reduced price—Advt.).

It should now be clear how Example 2 of our earlier test was encoded, and we give its beginning with the significant code elements circled and labeled.

Example d

If the same decoding principle be applied now towards the solution of Example 1, its opening is solved thus:

Example e

We then discover its complete message to consist of an alarmingly symbolic drama portraying half an Austrian soldier hammering together a darkened blue flypaper stage-set on a threatening blood-red moon, while a ghastly nine-year-old voice sprechstimminates:

In eighteen [hundred and] forty-one
I put my cord'roy breeches on,
I put my cord'roy breeches on
To work upon the railway

Manifestly the potentials of cipher composition are superior to those offered by competing synthetic techniques which share its non-musical objective and motivation, but so ineffectually. Furthermore, the chances for an inadvertently musical result are also considerably higher, even though the code technique meets fully the primary criterion of non-music by being forcibly constrained or organized about a musically irrelevant construction, namely the coded verbal message. Composition by encoding is far more flexible than isorhythmic, isomorphic, isotopic, isolated, serial, tension-gradient or timetable constructivist procedures; yet, like these, its principle excludes the chief condition of the art of music proper, which is that its processes be perceptible, intelligible and enjoyable as heard by human beings.

Of course the naive music lover may still ask, why waste time on any type of non-music? To this the experimental composer has ever audaciously replied, why not? And for him that is a very good answer, and for us a very great boon, so long as the antisocial impulse and result does not require our being cajoled into listening to it, or our acceptance of its precepts as either aesthetic or scientific.

But in exploring thus the further ramifications of the code principle we stumble upon a much broader and much more heady implication. For unlike the known outcome of the more primitive non-music techniques currently practiced, who knows? There might even be music in it!

Theorists of non-music teach that whatever is conceivable by the sovereign will of the composer is automatically true. It is quite conceivable, and depth speculation can never finally disprove it, that secret codes similar to our sample one have played a hitherto undiscovered role, not only in the procedures of medieval scholastics who had their guild privileges to protect thereby, and not only among their new imitators who delude themselves as to the novelty of the pastime, but also in the allegedly pure formal constructions of the standard classical composers.

It is not beyond reason, or at any rate not beyond what Die Reihe mistakes for reason, that such paupers in musical talent

as Bach, Mozart, Beethoven and Schubert were all badly in need of some rickety synthetic composition system, like the above-mentioned stage-set, to cover over their lack of ideas or impulses; particularly Bach, who had not even the advantage of speaking Viennese. So daring a notion is further supported by certain partial evidences of hidden intricacies of workmanship in classic compositions, which nevertheless manage to sound as if they consisted merely of music. These evidences are of the nature of deeply buried urlines, symbolic key-choices, motif-transmutation formulas, astonishing rediscoveries of the C major triad, Irish influences, and numerous other wondrous symptoms which enterprising rewrite biographers, educational predigestion experts, appreciation mongers and, alas! even certain music theorists, have already scavenged from the defenseless and forever undefinable subconsciousnesses of these composers; or so we find reported.

Now it should be apparent that these triumphs of analysis, won through arduous contemplation and tortuous translation from the German, must represent no more than occasional, superficial and fleeting traces of the true means by which the composers named blithely imposed their secret constructs upon supine audiences. We may not pursue further this revealing hint; the prospect is fraught with such terrifying implications that more than one pet theory would be destined for oblivion. For if classical composers did indeed employ a code practice, then no structural analysis on these models will ever produce a useful outcome; since, as we have effectively demonstrated, composition codes constitute perfect ciphers that can never be solved.

Those who boast of having made off with magic cipher keys while being anointed shall henceforth have to improve on their spiritualist credentials. But even such a showing has its consolations, in that the game of attributing to honest musicians those ingenuous and tangled fancies which haunt only certain speculative commentators may yet be brought to a halt, the composers in question belatedly granted their psychiatric clearance papers, and the possibility of both enjoying and understanding music without the intervening probings of note squeezers restored to weary lovers of the art of music.

Thus if Bach et al ever toyed with non-music codes, I, for one, will never give them away. But I don't think so, for they were quite sufficiently engaged in making music; though Mozart's interest in billiards and in dice has also been reported. Code

composition may occasionally be mistaken for music, or vice-versa, by the uninitiated, or at least those among them who have never heard non-music but have only come upon it in self-adulating manifestoes. The mistake, as I have previously suggested, hinges first on the confusing double reference of the term composition, and second on the disguise of many non-musical constructions in the semblance of musical notation; both phenomena that are more frequently encountered in association with the art of music. With all the contrived imitations and buffered foolishness abroad these days, people should be more wary, and should not allow themselves to be shamed into acquiescence or to be overawed by fad or bluster. If the king be naked, let us say so.

Apart from the special transcendent properties of the Thirteen Tone System (in The Music Review, XXII (1961) 152-171), no non-music technique can match the practical usefulness of the notation cipher. Its advantages for business communications are obvious, since complete privacy is guaranteed by the manufacturer; that same privacy which seems to have become the single agreed ideal of the tribe of composers, as they clamor to inform the public. Composition by encoding also has many accessory uses, as for broadcast station call letters, for doorbell or telephone-bell rings, for the tuning of automobile horns to the names and registration numbers of their owners, for distress signals of opera stars or for the writing of detective stories. Let us but consider the irresistible subliminal power of a singing commercial, or a jukebox rock, whose musical notation itself shall convey instantly and unmistakably the literal and exact message of the loudsell beer blurb or the softsell Junemoon lyric, and it becomes clear that all the more limited and obsolescent techniques for composing non-music, still being practiced in remote localities, ought to be forever abandoned or traded in. Not a listener need remain to hear their bitter end. For if a composer indeed proposes to engage further in such elegant recreations as the concoction of non-music, he might as well perpetrate it in full awareness and avowal of his aim and by the most modern and efficient code known.

* * *

* * *

FORUM

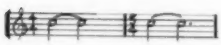
BRUNO BARTOLOZZI

The natural evolution of musical language and the spread of new techniques of composing have brought about certain deficiencies in the present system of musical notation as it has been handed down, substantially unaltered, from the beginnings of the 19th century to today. Such deficiencies concern particularly those very signs which, with the name of "notes," are the basis of the present system of notation: signs incapable of expressing with single notes the complete range of integral values. From this inadequacy was derived the distinction between "integral values" and "composed values," these last being the result of the sum of two or more notes joined for effect of the so-called tied values.

This arithmetic conjunction of different values to express only

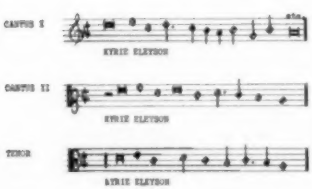
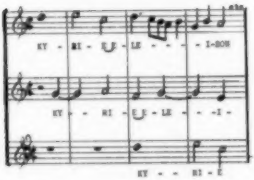
one can be reasonably construed not as a mere graphic expedient, but instead as a fact capable of distorting and falsifying the significance of a particular rhythmic articulation. If today, in fact, to express a whole note one uses two half notes tied together, this would appear altogether illogical and gratuitous. The same is to be said for any values whatever resulting from a combination of notes tied together.

Example 1

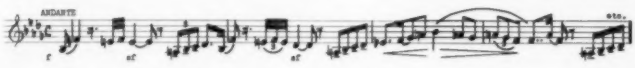


Evidence of how much the splitting of integral values can influence musical articulation may be found, for example, in transcriptions into modern notation of ancient polyphonic music, and particularly in those transcriptions where the introduction of regular bar-lines causes awkward splitting of values:

Example 2

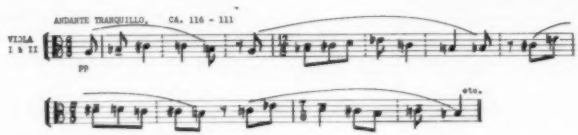
Palestrina: Missa Papae Marcelli

The necessity of devising a complete musical notation, and especially one more faithful to the very meaning of rhythmic articulation, became apparent the moment the composer chose to indicate the movement or section of a composition not by the "denominator" of the measure:

Example 3
Mussorgsky: Pictures from an Exhibition



but by its sub-division:

Example 4
Bartok: Music for Strings, Percussion and Celesta



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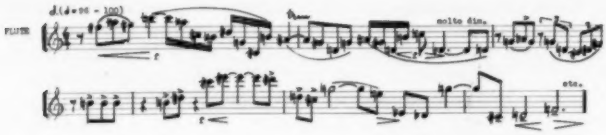
With this it was implicit that the component of each beat acquired identical rhythmic significance and that, no longer considering the existence of composed values, each rhythmic signature (be it $\frac{4}{4}$, $\frac{9}{16}$, $\frac{3}{4}$, $\frac{7}{8}$, etc.) had an identical function.

This is certainly not the place to identify the moment and the circumstances in which a different method of indicating rhythmic articulation came to be formulated. We are concerned here rather with emphasizing the necessity, for the composer, of employing graphic signs which assure, even to the standard of musical performance, the maximum accuracy. For us, this necessity is that each note, having the character and significance of an "integral value", can find its own suitable graphic sign.

We have said "having the character and significance of an integral value" to distinguish such cases from those in which the traditional notation can still respond completely to the rhythmic sense of the phrase.

Example 5
Schoenberg: Pierrot Lunaire, No. 7, "Der kranke Mond"

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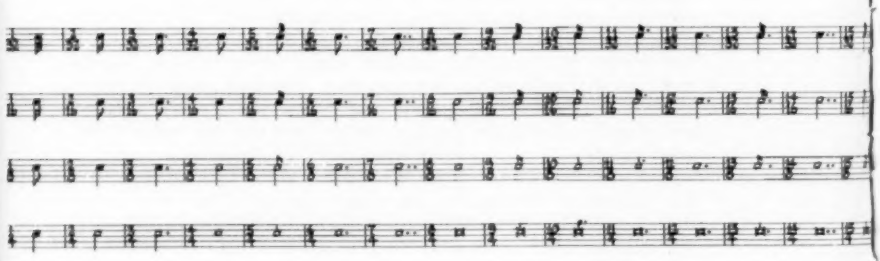
Moving on to expound the proposed solution, we shall say which are the necessary means to obtain it:

— to augment the value necessary for completion of a note, the five following symbols are adopted: and dot

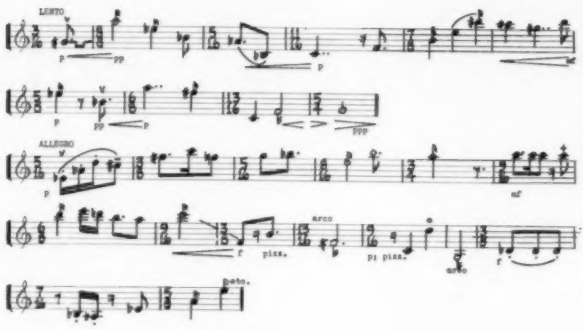
— the augmentation is applied above or below the note:

Consequently, the solution can be summarized in these terms:

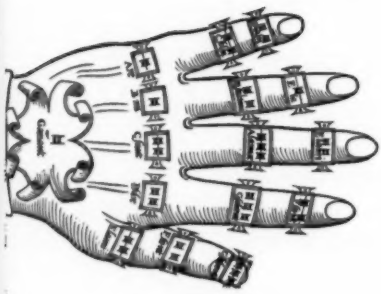
To obtain those values not obtainable with the present system of notation, one augments the value necessary for completion of the note by adding, above or below the note, the tail or the dot corresponding to the deficient value.

Example 6
COMPLETE TABLE OF THE PROPOSED NOTATION

A few examples of writing according to the modifications:

Example 7

Bartolozzi: Cinque Studi per Violino

Translated by Brooks Shepard

* * *



BOOK REVIEWS

STUDIEN ZUR SATZTECHNIK DER MITTELALTERLICHEN ENGLISCHEN MUSIK
by Ernst Apfel
HEIDELBERG, CARL WINTER UNIVERSITÄTSVERLAG,
1959. 107p., Notenanhang, 145p.

REVIEWER
NAN COOKE CARPENTER

During the past few decades, the music of medieval England has received a great deal of attention on the part of interested investigators; and the result of such study is a continually changing picture. Once considered largely peripheral to that of the continent, England's medieval heritage is now thought to have had its own strongly indigenous characteristics, independent of continental influence. Such a view is strengthened by this latest work of Ernst Apfel, a publication of the Heidelberg Academy of Sciences (of which Thrasybulos G. Georgiades heads up the musical division). The volume appears in two sections: Teil I, the essay, and Teil II, the musical transcriptions.

In his introduction, the author underlines the special value of this work: it brings together, he says, in one collection a

large body of music (13th to 15th centuries) from many different manuscript sources discussed up to now only in bits and pieces and never before transcribed and printed. In fact, most studies of English music, we are told, deal only with the two best known collections — the Worchester Fragments and the Old Hall Manuscript — leaving quite untouched numerous unpublished fragments from the 14th and 15th centuries. Studies of the motet, the most important single form of medieval music in England as on the continent, have similarly neglected this form in England, since English sources were either unknown or inaccessible to the investigator — for instance, to Professor Ludwig. Handschin and Dittmer are given credit here as being real pathfinders in a new approach to English music — Handschin in his observations about pieces of English origin in the Notre Dame Codex W1 and in the Montpellier Manuscript, and Dittmer in his work on the Worchester Fragments. Largely rejecting Bukofzer (whose views about continental influence upon English music he believes to be outmoded), Dr. Apfel states that his work rests heavily upon earlier studies made by Rudolf von Ficker and Georgiades. The introduction concludes with a catalogue of all manuscripts and manuscript fragments mentioned in the text; a list of editions and literature on the subject with music; and an alphabetical listing of incipits of the pieces to be discussed.

The textual discussion (Teil I) is divided into three chapters, the first two dealing with earlier music: I, music from the Worchester School to the Old Hall Manuscript written in parts; and II, the same music written in score. Chapter III deals with Old Hall music and pieces related to it.

Chapter I opens with a catalogue of all manuscripts under discussion, giving present location of the manuscript and listing individual pieces therein — with a footnote commentary and concordance. Following this is a detailed stylistic analysis of the music, in the light of which Dr. Apfel draws a number of definite conclusions. For example, he sees two different techniques of composition emerge for conductus and sequence in the Worchester collection: a motet-like technique, characterized by independent pauses in the voices; and a proper conductus-technique, characterized by simultaneous resting places in the various parts. Harmonically, Dr. Apfel also differentiates two stylistic traits: a sort of free chordality on the one hand and parallel movement of voices in five-three chords on the other. But the longest part of this chapter, naturally, is devoted to the motet. All motets in the collections are closely scrutinized — two-part, three-part, four-

part; and various interrelationships among them are thoroughly discussed. This section closes with a list of typically English characteristics in motets found in continental sources.

Chapter II deals chiefly with conductus, sequences, hymns, and pieces from the Ordinary of the Mass — pieces noted in score. And, again, the author begins by listing all compositions under discussion, giving the necessary information about the manuscripts in which they exist. Here there are two large divisions: (1) pieces with no observable cantus firmus, and (2) pieces with some form of cantus firmus. Trope, conductus, sequence, gloria, kyrie, credo come under the first listing. In the second division, pieces are listed according to the position of the c.f. among the various voices — again, chiefly conductus and pieces from the Ordinary of the Mass. As in his earlier chapter, Dr. Apfel proceeds here into a detailed analysis of the music, from a strongly harmonic point of view. Emphasis, in fact, is upon the various types of cadential structures and the development of the six-three technique. All this is frequently related to treatises of the period dealing with the same subject.

In Chapter III, Dr. Apfel rearranges and condenses the six categories set up by Bukofzer (Studies in Medieval and Renaissance Music) as a guide to his discussion of Old Hall and related compositions. He first lists the pieces to be dealt with under four headings: (1) the conductus-type (without cantus firmus); (2) the same, with some form of c.f.; (3) the "free piece" without c.f.; and (4) the motet-type with c.f. (tenor c.f. in long notes; isoperiodic; isorhythmic). A close analysis of the various compositions follows — with frequent reference and comparison to similar works by Okeghem, Dufay, Binchois, and others. In the light of this scrutiny, Dr. Apfel finds the "klanglich-freie Diskantsatz" especially characteristic of the motet of this period (with which he has dealt in an earlier essay, Archiv für Musikwissenschaft XII, 1955). He feels, indeed, that phrases characterizing this music have been, in the past, far too nebulous; and he proposes that such terms as "klanglicher, freier und klanglich-freier Diskantsatz" be used in the future, as more accurately descriptive. On the continent, he tells us (p. 104), the "klanglich-freie Diskantsatz" appears only in the Mass — never in the motet, which is limited to the "klanglich-freie Conductussatz."

The chapter ends with a brief summary, emphasizing again the independence of England's medieval music. Numerous freely composed early motets and rondelli in English manu-

scripts, closely related to the conductus, testify to this — types, says Apfel, found seldom on the continent, never in France. That English music was the first to incorporate the three- and four-part piece as basis for the new chordal technique after the Notre Dame period; that these motets were composed from the beginning in three or four voices (and not the result, as on the continent, of the practice of "Hinzufügung"); that the conductus, trope, and sequence continued to be cultivated polyphonically into the 15th century only in England; that the arrangement of a whole cantus firmus in conductus-like composition appeared almost nowhere except in England, related, of course, to the characteristic faburdon-practice — these are some of the conclusions with which Dr. Apfel supports his thesis that the music of England, from the 13th to the 15th centuries, was a native phenomenon, largely independent of continental influences.

Teil II contains 16 facsimiles of manuscripts in the Bodleian Library, the British Museum, and Magdalen College, Oxford, together with musical transcriptions of 59 pieces discussed in Teil I, none of them ever before published.

Some will agree with Dr. Apfel's cogent arguments and others will undoubtedly think his conclusions much too strong. The graduate student newly entering upon his work will wish for less compression and the fuller documentation that American and English scholars take for granted in books of this sort. (How many, for example, can identify the "Scottish Anonymous" mentioned on p.83?) But anyone with a genuine interest in England's musical heritage from the Middle Ages will find this a most illuminating study, a worthy companion to such recent publications as Frank Harrison's Music in Medieval Britain.

THE EVOLUTION OF TWENTIETH CENTURY HARMONY

by Wilfrid Dunwell
NOVELLO, London, 1960. 240p., 35s.

REVIEWER
ELLIOT WEISGARBER

The author, currently Dean of the Faculty of Music at the University of London was born in 1902 at Leeds. He holds a B.A. and Mus.B. as well as a Ph.D., awarded by the University of London in 1953. Much of the work in this book is based on material presented as a dissertation for the degree.

Any book of this nature written in 1960 is bound to find itself in stormy seas. But Professor Dunwell has tried to steer a steady course between the Scylla of the post-Webern-electronic school on one side and the Charybdis of orthodox dodecaphony on the other. He takes the conservative view; Bartok, Hindemith and Stravinsky (B.C.— before Craft) are his heroes. This point of view is expressed clearly in a single paragraph appearing in the conclusion of the book:

> Between the extremes of the expressive and nonexpressive, the emotional and the intellectual, the central stream flows on as always, the opposite qualities showing themselves in varying degrees of fusion and separatism. For the present writer, the composer who points the way most clearly in this direction is Bartok. He investigated new forms of order, his instinct as a musician gave them significance, whether heard purely as sound or viewed as symbols of thought and feeling. This has been the composer's function in all ages. At the present time, the point to be stressed is the new forms of order; and by using them imaginatively yet keeping a middle course, Bartok ensured his place in the line of succession and inspired confidence in his authority as a guide into the future.

It is this term "middle course" which characterizes the entire book. The author shows little or no enthusiasm for the twelve-tone technique. The only examples in the book are a few drawn from the Fourth String Quartet of Schoenberg.

In his introduction, Professor Dunwell defines the objectives and sets the limits to the scope of his study:

> It places emphasis on point-to-point details of technique and excludes the consideration of complete works and personal styles. The purpose is not to add to the large body of critical and historical writing which already exists, but to try and answer the question "Why does twentieth century music sound so unlike earlier music?"... The objective is limited, and even so, not completely attainable, since creative thought often reaches beyond existing experience and in so doing makes new channels for its expression.

The book is divided into two main parts. The first deals with the emergence of early twentieth century harmonic thinking from the practices of the previous century. The chapters are brief. They cover the appearance of the fourth, the augmented triad and the whole tone scale as building materials; new juxtapositions of chords; decoration of familiar chord structures; decoration as a structural function in itself. There is a chapter dealing with the effect of stylistic problems arising from dramatic situations, and so forth, on the choice of harmonic materials. There is a wide selection of examples in these chapters ranging from Wagner, Debussy, the early Bartok and Schoenberg to Hindemith. This entire section, then, is concerned specifically with the transition from what the author calls traditional harmony and the subsequent emergence of new tone colors and potentials inherent in the twelve tones of the chromatic scale.

The second part of the book is devoted to developments which are more removed from earlier traditions. First, the element of parallelism is explored. As a way of associating the available tones of the chromatic scale, the author regards it as having "a special technical utility of organizing progressions between chords". The expected examples from Debussy, Ravel and Vaughan Williams are here; but the Second String Quartet of Bartok is singled out for attention.

A chapter called "Pattern" follows. It is concerned with chordal and motive patterns, canon, and so on. "Pattern," says the author, "serves a double function: it controls the general formal structure and contributes to the details of harmonic textures. In these two aspects, pattern is an active force in later periods of technical formation".

A chapter on melody considers the nature of modality and its usage in modern techniques. There is a brief account of the

twelve-tone technique and an analysis of some short examples drawn from Schoenberg's Fourth String Quartet. There is also an account of Hindemith's views on the matter of melody as stated in The Craft of Musical Composition. The author also discusses melodically based music which owes allegiance to neither traditional key systems nor the twelve-tone technique. Examples of the music he likes — that of Bartok and Rawsthorne, and Schoenberg's Pierrot Lunaire — are cited here.

An all too brief chapter on the power of rhythm as a shaping force in harmony acts as an afterthought to the chapter on pattern. This needs much more attention.

Five chapters on various types of chord structure follow. They are well organized and should be of interest to the beginning student of modern techniques. The chapters called "The Inter-Relation of Melody and Harmony," and "Melodies in Combination" have important ramifications entirely germane to the central problem of the book which have been overlooked by the author.

There is a general index and an index of musical examples.

It is not an important book; but it will be of some value to the novice — student and amateur alike — who faces the difficulties of analyzing some of the music of the past 70 years. The author unfortunately has not avoided many things which are obvious and even trivial. It is clearly not a textbook and must be regarded as a preliminary essay for a far more detailed study.

TWENTIETH-CENTURY HARMONY: CREATIVE ASPECTS AND PRACTICE
by Vincent Persichetti
W.W. NORTON AND CO., 1961. 287p., $5.45.

REVIEWER
J.K. RANDALL

Vincent Persichetti and the W.W. Norton Company have issued a grab-bag of Mr. Persichetti's recipes for short-order cookery under the label "Twentieth-Century Harmony". By Mr. Persichetti's own admission, the book may be used as a "text" in "advanced" harmony courses.

The relevance of Mr. Persichetti's recipes to the music of this century is forcefully suggested by the fact that not a single one of them is attributed to any particular composer or group of composers. Indeed, not a single piece of music, or a single composer or group of composers, is mentioned so much as once in the entire body of the text. All references to pieces of music are appended to the various chapters of the text in the form of lists, whose frivolity is apparent in the captions which head them, in the kind of citation which they include, and in their scarcely credible omissions. In the earlier chapters the captions are quite specific: "Two-part writing in a two-voice work" (first item: Bela Bartok, 44 Violin Duets), "Two-part writing in a work of more than two voices", "Pentatonic writing", "Whole-tone writing", "Various kinds of seventh and ninth chords", "Fifteenth and seventeenth chords", etc. In general, the citations under these headings refer to a particular spot in a piece. For example, we may verify that somewhere in the second movement of Bartok's 1st Quartet the first violin plays a whole-tone scale; or that the first nine attacks of Debussy's Prelude 5 (Vol. I) state five different pitches (caption: "Pentatonic writing"). In such references, Mr. Persichetti is concerned exclusively with the presence of some collection of pitches for which he has a name. Later on, the captions become more whimsical: "Harmony with characteristic doubling, spacing, or omission", "Irregular harmonic rhythm", "Passages featuring repeated notes or chords", "Passages with characteristic dynamics and rests", "Works containing contrasting tech-

niques", "Passages containing various kinds of harmonic textures", "Unique thematic ideas", etc. It should be stressed that throughout the book these captions are the sum total of Mr. Persichetti's comments upon the lists of works which they head. Equally symptomatic of the educational value of these lists is the absence of Debussy's name from the lists headed "Real parallel harmony" and "Tonal parallel harmony"; of Le Sacre du Printemps from the lists headed "Examples of pedals" and "Examples of ostinatos"; and of any reference to a twelve-tone work of Schoenberg, Berg, or Webern from the lists following the subchapter entitled "Serial Harmony". On the other hand, Schoenberg's Violin Concerto is listed elsewhere under the heading "Chromatic harmony with chromatic melody": the reference is to p.33 of the piano reduction. I submit that the advanced student might even survive exposure to the first page and that the advanced student who reads music might risk a peek at the orchestral score. (Curiously, although Mr. Persichetti professes to regard harmony and its instrumental setting as inseparable, his lists repeatedly cite the piano reductions of works for which the orchestral scores have long been available.) The very next work in the same list is Scriabine's 9th Sonata. The fact that the Schoenberg and Scriabine works are both based upon rather specific principles of pitch-organization, and that these principles are radically different for the two works, does not interest Mr. Persichetti. The main thing is that both pieces splatter sharps and flats all over the place.

A sampling of Mr. Persichetti's recipes cannot fully portray the dense musical and linguistic fog which seals off the world of the text from the world of the lists:

Pizzicato strings define uncertain passing tones in woodwind voices, and a harp may underline obscure rhythms in lagging strings. (p.27)

Chords by fourths are used as "dominants" in cadences of any harmonic idiom. (p.101)

Ornamental tones increase harmonic circulation in passages of clusters. (p.132)

The harmonic ambiguity or sudden unison allows for the entrance of any texture. (p.273)

The versatile ornamental tone also provides textural means for entering any harmonic region. (p.273)

Mr. Persichetti courageously demonstrates his hundreds of

recipes with the assistance of musical fragments concocted for the occasion, I can only assume, by Mr. Persichetti himself. In this attempt to compose-out a half-century, Mr. Persichetti has exceeded his capacity.

Although the few theoretical ideas which do seep through the prevailing shroud of obscurity† are in large part already familiar from Hindemith's Craft of Musical Composition, Howard Hanson's Harmonic Materials of Modern Music, and from the most benighted traditions of chord-pushing pedagogy, no credit is given to any of these sources. Indeed, no book about contemporary music, nor any article, nor any book or article about any musical subject whatever, is cited at any point in Mr. Persichetti's book.

Of the following, however, Mr. Persichetti is undoubtedly the sole proprietor:

> Dynamics have a rhythm that is projected by means of piano, forte, crescendo, diminuendo, sforzando, and subito directions of accentuation. (p.226)

In short, dynamics have a rhythm that is projected by means of dynamics. The book abounds in such tail-chasers. On p. 265 we have this one: "When a melodic set includes one or more identical notes, doubling of chord members produces colorful serial doublings". Mr. Persichetti gives no example of a set with one identical note, although presumably even in this case doubling would produce doubling. Less original, but of some sociological interest, is Mr. Persichetti's impression, as of 1961, that the twelve-tone system is a "contrapuntal principle" best dealt with in a "treatise on counterpoint". (p.262) In accord with his cover-all-exits approach to controversy, Mr. Persichetti does concede in the next paragraph that "when harmony is regulated by a horizontal, unifying idea (twelve-tone or not), the texture may be serial; this kind of writing creates harmony of extraordinary compactness through the manifold variations of the motif relationships". The clause preceding the semicolon means that serial principles produce a serial texture — with the term "texture" enjoying syntactical rather than semantical status. Here and throughout the book Mr. Persichetti stands unwaveringly on guard

†W.W. Norton's editors were gratuitously tolerant in committing to the printer such expressions as "ease of readability," "played on a medium," "when color gradations of the ninth formations are made part of the composer's aural apparatus," etc.

against any unseemly emphasis upon those techniques of twentieth-century composition usually associated with that number of notes which is greater than eleven but less than thirteen. Although Mr. Persichetti hazards an excursion into serial cookery on page 263, he serializes only seven notes and thus emerges with his virtue intact.

In view of Mr. Persichetti's notion that whatever can be done at all can be combined with everything else, he naturally neglects to point out that much of the best-known music of the twentieth century — specifically, works of Bartok, Copland, Debussy, Hindemith, Satie, Schoenberg, Scriabine, Stravinsky, and Webern; not to mention Boulez, Cage, Carter and Stockhausen† — has been consciously and often explicitly produced to elucidate "new" or "extended" premises conceived as antidotes to allegedly outworn traditions or inadequate alternatives. Mr. Persichetti's version of the history of the past six decades appears on p. 11: "Composers have, in their music, coordinated the various musical resources of the early part of the century." (This is Mr. Persichetti's only hint that the twentieth century may have "parts".) Now surely any composer who has "coordinated the various musical resources" exhibited in the works of Satie, Scriabine, and Webern deserves, at the least, to be mentioned by name. Or on second thought, if by "coordinated" Mr. Persichetti means "joined by a 'sudden unison' or a 'versatile ornamental tone'"; and by "musical resources," scales and chords; then perhaps we should allow the identity of that composer to remain an open secret between the author and the advanced student.

†All of these composers are cited in Mr. Persichetti's lists.

DISPOSITIONES MODORUM: DIE TONARTEN IN DER MUSIK PALESTRINAS UND SEINER ZEITGENOSSEN

by Siegfried Hermelink

H. SCHNEIDER, TUTZING, 1960. 194, 44p. DM 25.65

HENRY W. KAUFMANN

The title of this volume, the fourth in the series of the Münchner Veröffentlichungen zur Musikgeschichte, is taken from the preface to Heinrich Schütz's Geistliche Chormusik. The ordering, classification, and attributes of the various modes which Schütz considered prime requisites for composition form the basis of this study, but in terms of the 16th century. Such a task is complicated by the theoretical observations on the modes found in most 16th century treatises, which treat the subject mainly from the point of view of monophonic Gregorian chant. Dr. Hermelink's aim is to relate these modal concepts to polyphony, and as a first step in this direction, he finds a direct correlation between clef-combinations (Schlüsselung) and the treatment of modes in actual compositional practice. In his opening chapter, he classifies the combinations found in the works of Palestrina into three groups: (A) plena voce, (B) mutatis vocibus and (C) paribus vocibus. Group A, 91% of the total, is further subdivided into "hohe Schlüsselung" (Soprano=G2, Alto=C2, Tenor=C3, Bass=F3 or C4) and "tiefe Schlüsselung" (S=C1, A=C3, T=C4, B=F4 or F3). Group B, only 4%, is also divided into high and low combinations, considered as variants to Group A, since only the soprano is affected: high=C1, C2, C3, F3 or F4 and low=C2, C3, C4, F4 or F3. The third group, C, written for equal voices and comprising only 5% of Palestrina's work, uses part of the normal system (Group A), depending on whether high or low voices are featured. It will be noticed that Dr. Hermelink rejects the common classification into chiavi naturali (C1, C3, C4, F4), high chiavette (G2, C2, C3, F3) and low chiavette (C2, C4, F3, F5) which is actually of much later provenance than the 16th century, dating probably from Paolucci's Arte prattica di contrappunto (1765-1772).

In Chapter II, the author attempts to prove that, by the time of Palestrina and his contemporaries, the choice of clefs was no

longer an arbitrary whim of the copyist but an integral part of the score (Partitur), decided upon by the composer himself. The use of high or low Schlüsselung often is dependent on the particular modal form of the cantus firmus. Since contemporary theorists offer little support for a new concept of modality on a polyphonic basis, Dr. Hermelink, in Chapter III, turns to the music itself for his empirical facts. His method presupposes the varieties of clef-combinations (Chapter I) and the compositional significance of the score (Chapter II). He then outlines the following technical factors which contribute to his new concept:

1 Grundton. This basic tone is conceived as a sort of axis around which the composition revolves. It is not so indefinite as the modal finalis, but neither is it so fixed as the tonic in the harmonic system. It serves as a goal-tone toward which all the other melodic tones gravitate.

2 Melodiebildung. The problem of ambitus (generally not exceeding 20 tones from bass to soprano) receives a new significance if allied with the notion of Grundton and Schlüsselung. Dorian pieces, for example, are found with high and low clef-combinations, "in loco nativo", i.e., without signature (Grundton=D); and in a transposition of a fourth with the signature of one flat (Grundton=G). These differ markedly in melodic structure, since, in all four cases, the Grundton is located in various places on the staff and will hence control the direction and position which the other tones will take. The melodies of all the other modes will similarly be based on the Grundton-ambitus relationship, resulting in a series of predictable melodic patterns, and replacing the older concept of authentic and plagal.

3 Cadences. These are basically achieved by two voices, not chords (as after 1600) which come together in a unison or an octave. This tone usually emphasizes steps of importance which differ from mode to mode, although special importance seems to be placed on the fifth degree. The influence of Schlüsselung is less in evidence in connection with cadences.

4 Klang. Sonority, though not dependent on chords in the harmonic sense, is nevertheless the result of interval combinations which create triad-like formations. Dr. Hermelink recommends the term sonority-relationship (Klangverhältnis) rather than chord for such structures. The sonority is directly influenced by the position of the soprano note (8, 5 or 3) in the complex, and is thus dependent on the ambitus-Grundton

relationship, and hence to Schlüsselung.

In his fourth chapter, the author discusses the relationship of modality to tuning and comes up with the proposition that all the modes written in high Schlüsselung actually sound a minor third lower. There also exists a connection between the clef-combinations and the ethos of the modes. Compositions with high Schlüsselung are livelier and happier than the more serious pieces in low Schlüsselung. This concept differs from the Affekt usually assigned to the individual modes.

In his final chapter, Dr. Hermelink analyzes in detail a select group of compositions which illustrate the main points of his thesis. These compositions are incorporated into a Beispielsammlung which is offered as an integral part of the book.

The chief difficulty with this study lies in the attempt to represent Palestrina as the typical composer of the 16th century. Despite the promise of the title, the only contemporaries mentioned more than in passing are Marenzio and Lasso. The former is represented by a cursory analysis of a single composition contained in the Beispielsammlung; the latter, by references to his more conservative, basically diatonic, works. Almost no attention is paid to the striking innovations of the Venetian school, which are of prime importance in any study purporting to discuss the interim stage between modality and tonality.

Dr. Hermelink's classification of the clef-combinations used by Palestrina is quite helpful, but his desire to relate these combinations to the observations of contemporary theorists leads him into trouble. This is especially true in the case of pieces "a voce mutata". Vicentino, whose treatise is completely ignored in this study, describes such compositions as written "without sopranos" and advises composers to "watch that the extreme ranges do not exceed fifteen tones [two octaves] and at most, sixteen with the semitone". (N. Vicentino, L'antica musica (Rome, 1555) f. 84v (incorrectly numbered 79)) Compare this with Morley's remark, quoted by the author, that "musicians also used to make some compositions for men only to sing, in which case they never pass this compass: [the example which follows is limited to a range of two octaves]". (T. Morley, A Plain and Easy Introduction to Practical Music, ed. R. A. Harman (London, 1952) 275) Vicentino explains that the mutation was generally accomplished by reading the soprano part an octave lower, thus transforming it into a tenor, and that it was also possible on occasion to convert a tenor

part into a soprano by raising the tenor an octave, although this practice seems to have been less common than the reverse procedure. (Vicentino, op.cit., f.92v-93) Yet Dr. Hermelink, unaware of this detailed explanation, equates the Morley quotation with the concept "paribus vocibus" instead of "mutatis vocibus".

The author is on firmer ground when he states that the clef-combinations are indicated in the score by the composer himself. (Not all scholars are agreed, however, that the "tabula compositoria" of Lampadius, to which the author refers, can be equated with score. Cf. the review by J.A. Westrup of A Musicological Offering to Otto Kinkeldey, in Music & Letters, XLII (1961) 281-283, esp.282) Whether these have any real compositional significance is still a moot point. Most 16th century vocal music depends in great part on the nature of the text, and the purely technical features of composition are often subservient to textual demands. Granted that Palestrina is more conservative than many of his contemporaries (he is about as typical for the 16th century as Richard Strauss would be for the 20th), even in his music, the technical categories established by Dr. Hermelink do not always work. The melodic formulations established by the ambitus-Grundton relationship are not nearly so apparent in the bass part as in the other voices. The author recognizes the existence of symmetrical sequences of triad-like formations and series of descending fifths in the music of his favorite composer, yet will not admit the possibility of real modulation, despite Willaert and his school. Cadences on practically every scale degree can be found, even in the music of Lasso which the author has not cited. For the affective side of the compositions that he is examining, the author relies too much on his experiences as a choral conductor and his subjective reaction to the music. Zarlino's characteristics for the twelve modes, given in Appendix B, and the comments on the various modes in Adam Gumpelzhaimer's Compendium musicae (1591) are indicated, but since these observations refer mainly to the tenor, they are modified by Dr. Hermelink in the light of his own experiences. Vicentino would again have been of help in this matter, since in his treatise, the ethos of the mode is first presented for the monophonic form of the cantus firmus and then for the entire polyphonic complex.

On the question of transposition down a minor third for pieces in high Schlüsselung, the author is on especially weak ground. Even he admits that no contemporary theoretical justification for such a transposition can be found, yet this does not pre-

vent him from evolving a terminology entirely derived from his arbitrary concept. (For example, B-Dorian, i.e., written D but sounding B because of the high Schlüsselung).

The scholarly apparatus employed by the author is impressive, but the work as a whole lacks the security of unquestioned evidence. Nevertheless, it serves to point up the need for more studies in this vexing area between Gregorian modality and major-minor tonality. Despite his Procrustean proclivities, Dr. Hermelink has offered many provocative insights into a problem which, it is hoped, will find a more fruitful solution in his own future work.

THINKING FOR ORCHESTRA

Practical Exercises in Orchestration
by René Leibowitz and Jan Maguire
G. SCHIRMER, INC., NEW YORK, 1960. 240 p., $6.00

REVIEWER
KENT KENNAN

Intended for the student who has already acquired a basic knowledge of orchestral instruments, this book proceeds on the following plan. In the first section the student is given reductions of short excerpts from orchestral scores of various periods along with brief comments designed to help him in scoring intelligently. He then orchestrates these reductions. Finally, he compares his results with the original scorings which are contained in the second section of the book along with further comments. This type of exercise is of course not new; but this is the first time to my knowledge that it has served as the point of departure for a book. Thinking for Orchestra makes a valuable contribution in providing numerous reductions (ranging from one to twelve staves in format) of orchestral music past to present. Ranging from Haydn to Sessions 23 composers are represented, and the

styles of orchestration are divided into four categories: classical, early romantic, late romantic and modern. As the title suggests, the chief aim of the authors is to train the student to think directly in orchestral terms. Furthermore, they hope to free him from a dependence on orchestral formulas (sometimes of doubtful validity) culled mechanically from texts.

The chief question bound to arise in a consideration of this method is: What conclusions should the student draw from the differences between his scoring and the original? Is his way necessarily "wrong" at the points where it differs? Might not the composer himself have scored the same passage differently in another context (or even in the same)? The authors' answer is that it matters not whether the student "is 'lucky' enough to come close to the specific instrumentation used in the original score .... The really essential part of the student's task is to understand the musical nature of the measures he is dealing with and to conceive these measures within the framework of a musically appropriate setting". A possible objection here is that not all students who work on their own will be able to distinguish between features which are actually inappropriate and those which merely happen to differ from the original. It would seem advisable that a teacher or seasoned orchestrator be on hand at this stage to discuss the differences and their significance.

An attempt to pass judgment on the effectiveness of the Leibowitz-Maguire book without testing it through actual use with students would be premature and unfair. But an initial acquaintance with it prompts these observations. First, the musical excerpts seem remarkably brief. (They average perhaps four to six measures but in some cases are as short as one measure). Such brevity is of course necessary if so many examples are to be included and if the book is not to assume impractically large proportions; but as a result the student orchestrates only a small fragment in each case, one which appears out of context with the larger scheme of the work or movement as a whole. Consequently the question of the relationship between phrases and sections of a work (certainly one of the most important considerations in scoring) never arises. Apparently the authors themselves regret this situation, for they suggest in an introductory note that the student "complete his studies through analysis and reduction and rescoring of many other works". The idea of analysis is obviously one which can only be seconded heartily. In fact, there should be a good deal of score study even before the student attempts the exercises in this book. But the suggestion that he reduce

the works that he will then re-score seems patently unrealistic; surely he will remember too much from the process of reduction to make the re-scoring really instructive. In a class or group this problem can be solved by having one student make a reduction of a score to be re-orchestrated by another student; both will then learn from the experience.

On the whole the choice of musical excerpts seems well made, with an eye to breadth of stylistic range and variety of texture (though the absence of any music by Bartok or Prokofiev might be mentioned as regrettable). Certain passages could be questioned on the grounds that their orchestral setting is likely to be familiar to students who are ready for this book, for example, the opening measures of Le Sacre du Printemps and the excerpts from the Beethoven Fifth and the Schubert Unfinished. It is true that even in familiar works one sometimes discovers on closer examination subtle touches hitherto unnoticed. But the question remains as to whether less familiar music might have been more instructive in some cases. The authors are to be commended for including some excerpts from operatic literature, from which students can gain practice in gauging the weight of orchestral sound against vocal parts. For the most part the musical examples are admirably accurate. The half-dozen or so errors that came to light are chiefly minor, an exception being the interchanging of two parts of an example on p. 122.

Observations about musical structure and points of scoring are generally well taken. The chronological approach works out effectively and gives the reader a good idea of the changes that occurred in the orchestra from Haydn's day to the present. The literary style, though not always distinguished, has the virtue of being sensibly informal, with the authors addressing the reader directly. On the debit side there is a tendency in the suggestions that accompany the sketches to include comments of such a general or vague nature as to be of no real value to the student, for example: "The second movement of Schubert's Unfinished Symphony. . .shows many interesting features of orchestral thinking". Part of the difficulty here is the attempt to give clues on scoring without revealing anything too specific, an attempt which sometimes seems ineffectual and hardly worthwhile. Thus we read: "An entirely new type of string writing has been employed here. Can you figure it out?". . . or: "Try, though it may be difficult, to understand the exact writing for the brass". Also of doubtful value are the repeated warnings that the exercise to follow may not be as easy as it appears (on p. 26, 29, 33, 44, 45, 60, 99, etc.).

However, these relatively minor objections do not weigh heavily in an appraisal of the book as a whole. Its uniqueness of approach, its broad historical scope, and its emphasis on first-rate music are among the qualities which should make it an extremely welcome addition to teaching materials. Given the time restrictions on most orchestration courses and the need for other kinds of exercises, such as actual transcribing from other media, it may not always be used in as wholesale a fashion as the authors apparently intended, but in any case it should be valuable in a supplementary role for use with students who have had at least a semester of orchestration (preferably more) and some score study. It should be especially valuable for students who are interested in the more creative aspects of scoring.

Let us hope, incidentally, that the book's users will have great strength of character. Otherwise they will almost certainly yield to the temptation to look at the original scoring before they have finished their own, and the whole point of the exercises will have been lost.

ACOUSTIQUE MUSICALE

a collection of papers by various authors
ÉDITIONS DU CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE. PARIS, 1959. 259p., Fr. 3400

REVIEWER
WAYNE BARLOW

A collection of scientific monographs is the least likely place one would look for the gratuitous nomination of the universal adoption of the Western musical scale as an aid to international understanding. But the suggestion is there, and roundly applauded by all the savants gathered to hear the latest news of researches into the nooks and crannies of musical acoustics. True, the suggestion is made during a question-and-answer period, but it is faithfully recorded along with everything else

of substance said during a three-day symposium (International Conferences of the National Center for Scientific Research, Marseilles, May 27-29, 1958).

The monographs bear upon only two general areas of the broad field of acoustics: (1) theories of scales and the meaning of harmony, and (2) the physics of stringed and wind instruments. It is made clear that the entire conference was to be devoted only to these considerations, and it was further expressly stipulated that papers were not to deal in any way with electronics in relation to music. The result is a collection of articles dealing with some of the traditional concerns of theorists, teachers, practical instrument makers, and musical scientists. If there is nothing included as inflammable as a discussion of concrete or electronic music, at least some of the papers are controversial enough to make interesting reading.

Perhaps the best way to convey some idea of the scope of the book is to list the titles of the papers. In the first area are the following:

1 Some new ideas on consonance.
2 A scale of 1/5 tones.
3 Evolution of the scale in Oriental music.
4 The problem of the seventh harmonic before musical history [a résumé only].
5 The role of fusion in judging consonance.
6 The influence of psycho-physical factors in the sensation of consonance and dissonance.
7 Reflections on some considerations inherent in musical acoustics.
8 The problem of simple ratios. The notion of psycho-arithmetics [psycharithme].
9 Some proposed reforms in acoustico-musical theory.
10 The vocabulary of acoustics as related to the evolution of the language of music.

The following papers deal with the physical properties of instruments:

1 Determination of the quality of instruments by means of the sound spectrum.
2 Investigation of the tonal spectrum of the organ.
3 The synthesis of sound of the pipe organ.
4 On the intonation of several musical instruments.
5 Acoustical examination of violins at the Physical-Technical Institute [at Braunschweig].
6 A contribution to the study of the cello.

7 Researches into the timbre of violins and guitars and into the initiation of vibration of a harmonium reed.
8 The excitation of a free-reed organ pipe.
9 Measurements of acoustic impedance.
10 A study of the stability of sustained oscillations in a vibrating pipe coupled to a quiescent pipe.
11 The spectrum of a bell tone.
12 Acoustical properties of horns and trumpets.
13 The physical characteristics of the pharyngo-buccal pavillion in phonation.

The nature of the articles in Part II is self-evident; most are reminiscent of similar papers contained in the Journal of the Acoustical Society in this country, and most are of interest only to advanced workers in the field who possess an adequate mathematical background. Special mention should be made of the third article, however; this is a practical discussion of organ pipe construction with many drawings and tables of measurements which leans heavily upon the work of Frobenius.

Since this reviewer was mainly interested in the material of Part I, and since it is obviously impossible to cover every article, the remaining space will be devoted to the articles dealing with consonance and scales. These papers are written from a variety of points of view, including the speculative, the historical, the phenomenological, the metaphysical, and the philosophical, and represent the results of current research and investigation into a wide variety of topics.

The first article contains a useful review of older theories of consonance, including some strange ones from the Near East (the author is an Iranian). In particular, the Persian, Avicenne, is quoted at some length for his theory of consonance based upon superparticular ratios (of the form N/N-1). Avicenne carries consonances of this type to the ratio 46/45! The "new ideas" heralded in the title have to do with a revised order of the intervals (from most to least consonant) based upon combination tones set up by the two notes of the interval. In laying the groundwork, the author sheds some light on the perennial argument over the subjective or objective nature of combination tones by citing the work of Stanislaus Dolinski at the Sorbonne in 1948; Dolinsky apparently was able to make a convincing case for the objective existence of combination tones, that is, outside the ear itself.

The second and third articles approach the problem of microtonal division of the octave from different directions. The

author of the second puts up a purely hypothetical argument for 1/5 tones based on the melodic difference C-D^{b} and C-C# (he calls them major and minor half steps). His argument convinces nobody and inspires the somewhat caustic observation from the floor during the discussion period that it is "no business of the scientist to tell the musician what to do." On the other hand, the author of the third article makes a brilliant impression with his exposition of the evolution of the Oriental scale. In his view, the scale is nothing other than a system based upon two ancient scales: the "melodic" scale of Pythagoras and the "harmonic" scale of Aristoxenus. And this turns out to be a scale of 1/5 tones. It is during the discussion period following his paper that the author (the Iranian author of the first article) observes wistfully that the universal adoption of the Western 12-tone scale should be an aid to international understanding.

The notion of fusion advanced by Stumpf as a measure of consonance is explored and extended to triads in the fifth article. The results are inconclusive, though, predictably, fusion does play a role. The sixth article, with exquisite logic and crushing weight of evidence, proves once again that the tolerance of the ear is an important factor in judging consonance and dissonance. In general, the papers have something new to offer, but the fifth is a classic example of what has long been observable in this country: a fondness for re-proving old propositions with increasing refinement.

The eighth article, dealing with simple ratios from a speculative and metaphysical approach, provides the most difficult going. It postulates first and second hierarchies of complexity of numbers and draws upon a concept that this reviewer has translated as "psycho-arithmetics." Freely translated, the latter is "the abstract representation, in mathematical form, of psychical phenomena."

The "back to Pythagoras" movement is eloquently argued by the author of the ninth article, who wants to see the Pythagorean scale rehabilitated and placed on an even footing with the tempered scale. The paper also contains an anguished plea for lowering the pitch standard to A-432. For this, the enthusiastic support of all segments of French musical opinion is claimed, with the exception of the instrument makers; the latter are unwilling, apparently, to forego their export trade in the United States for the sake of what the author hopes will be a new "metric" A. American jazz, he says darkly, will force pitch upward still more, and then where will we be? Al-

ready the symphony orchestras play C major symphonies in the key of $D^{\flat}$!

One of the interesting facts that emerges from the monographs and from the floor discussion is a clear awareness, sometimes sharply stated, that music as a science is not the same thing as music as an art. If there is no particular contribution to music as an art — and there is no reason to expect otherwise in a volume such as this — the book is nevertheless valuable for bringing up to date the state of man's thinking about aspects of music that have intrigued him over the centuries.

DIE ENTWICKLUNG DER MUSIKTHEORIE IN ENGLAND NACH DER ZEIT VON J.P. RAMEAU

by Erwin R. Jacobi
P.H. HEITZ, STRASBOURG, 1960. 2v., S.fr. 42.

REVIEWER
ARTHUR HUTCHINGS

This book was originally submitted as a thesis at the University of Zürich; it is now issued as v. 39 in the series of musicological studies founded by K. Nef of Basle. A translation of the initial chapter, appearing in v. 2 of this journal, November 1957, enabled its readers to examine the influence of Rameau upon English theory during Burney's lifetime and on to about 1820. In it Jacobi does not examine Geminiani's unfinished instructions on modulation nor his The Art of Accompaniament [sic] with a title similar to one of Rameau's publications; there was no reason why Geminiani should have been considered in this chapter since his sway in London preceded Handel's and his theoretical writings came after his retirement to Dublin. (See M.E. McArtor, Francesco Geminiani, Composer and Theorist. Unpub. diss. (University of Michigan, 1951)) I mention Geminiani only to support Jacobi's point that the Italians did not share the English fascination with Rameau's

doctrine of the "generating bass".

Had that fascination diminished after a Francophile epoch, such as that of Gottsched, Mattheson and Telemann in Germany, Jacobi's undertaking, though intrinsically valuable, would have been not only smaller but of far less interest beyond the strict limits of historical documentation; but the findings already translated initiate an inquiry of doctoral scope and unbroken aim because Rameau's doctrines "raged unchecked by taste or commonsense, and culminated in Dr. Day's application of homoeopathy to the art of music". (Tovey, Harmony. In: Encyclopaedia Britannica (1929 ed.)) Jacobi's work therefore culminates in his shrewd commentary upon Day's publication of 1845, to clarify which he provides in facsimile the whole of Part 2 of Macfarren's Alfred Day's Treatise on Harmony, edited with an Appendix (London, 1885).

Dr. Alfred Day was not a professional musician but a London medical practitioner of some social standing. To dismiss him as an armchair pseudoscientist is unjust unless we deal as harshly with Rameau. But Day attempted to classify on an acoustic basis a far wider vocabulary of chords than did Rameau and to bring them into a closer relationship; and he lived, of course, after men of his education knew the essentials of scientific method. In the words of Charles E. Stephens, author of the heaviest polemic against Day, "it is the ruling speciality of this theory . . . that any note which can be used in connexion with a particular key must have an absolute existence in that key, and that this applies not only (as is already taught) to diatonic, but also to chromatic notes". (On the Fallacies of Dr. Day's Theory of Harmony. In: Proceedings of the (Royal) Musical Association, February 1875.) In the words of the more diplomatic Parry, Day "considerably simplified" teaching "by classifying many chromatic discords under a single key system". (See Harmony in Grove's Dictionary of Music and Musicians, 3rd ed., 1927.)

The facsimile portion of Macfarren's edition of Day's treatise (subtitled Chromatic Harmony, or Harmony in the Free Style) contains the bulk of Day's excursions beyond Rameau. As Jacobi remarks, it is interesting that Day does not actually commend or, indeed, refer to Rameau's theory of a root-bass, but assumes that his English readers accept its application at least to diatonic chords in the major key. Though Rameau's mistaken zeal begins with the Génération harmonique (1739), a practical manual for the apprentice, it is in his more philosophic last work, the Origine des Sciences (1762) that, having

been driven to the 45th partial for tones on which to build all his chords, he can use a speculative treatise for declaration rather than for demonstration of his belief. Nature, he says, has endowed us with harmonic sense. He does not mean simply that we have inherited it from our later forbears who used chordal harmony and its notation, but that long before them melody itself had its origin in harmony of which the musician may not have been conscious. Day, on the other hand, seems to have had full confidence in his demonstration, so that when he could not relate the minor third to its tonic through partials of the dominant, supertonic or other degree, he simply dismissed it as an arbitrary creation of the mind or imagination.

Jacobi is to be commended for not deviating into scientific judgments. He may have more than a working musician's knowledge of acoustics but it is not part of his purpose to enter the field we associate nowadays chiefly with Dr. Llewellyn S. Lloyd. His concern is with the persistence in English teaching of theories of which there is no trace in, say, Riemann's books. Rameau's doctrines had either been disregarded or openly checked by the disciples of J. S. Bach, such as Kirnberger, Krebs and Kittel, by Bach's own sons, and by Lutheran cantors who learned harmony by playing continuo parts at the direct stimulus of figures and basses which were not considered as generators. (I myself doubt if they often wrote out chordal exercises; it seems to have been chiefly for instruction in counterpoint that the written exercise was deemed necessary.) One of Jacobi's most interesting sections shows that, early in the nineteenth century, the trend of English theory was similar, for he devotes several pages to A.F.C. Kollmann (1756-1829) a teacher little noticed by previous writers and erroneously thought by Schweitzer to have been a native of England. Kollmann prepared for publication an edition of Bach's WTC and actually issued analyses of the fugues. Like Dr. Pepusch, his predecessor in migration, Kollmann rejected Rameau and was bitterly attacked for doing so by M.P. King. In an age when literary rivals were not careful to hide rancor, their bickerings and others like them might never have been worth recording but for the appearance of Day's work and the prestige which it gained through the support of Sir George Macfarren, Cambridge professor and principal of the R.A.M., G. A. Osborne's reading of a paper about it to the Royal Society, and the enormous publicity that followed upon the discussion among opponents, supporters and partial supporters.

When one has recorded one's satisfaction in the thoroughness

of the compilation and shrewdness of the connective commentary one cannot further review a book like Jacobi's without inordinate lengths of quotation. Yet my limited acquaintance with American teaching (from books issued by the Eastman School, from works by university professors and such papers as Glen Haydon's at the 1939 I.C.M., which happens to deal with Day and his English critics) leads me to wonder if some American readers imagine that these are remote issues, already insignificant when Stanford, in his Musical Composition (1922), said that Day "irrigated a wide area of low-lying ground, and we are still suffering from the effects of its miasma". Perhaps, then, I may be excused for finishing in anecdotal style.

Jacobi tells us that he sent a questionnaire to leading English universities and colleges of music in 1955; the university musicians assured him that among them Day's theory was rejected; the conservatories were less forthcoming but suggested that among their harmony teachers Day's theories were obsolescent, or lingered in a much modified form. Yet in the very year of Jacobi's questionnaire I witnessed what justified the document. I had imagined that in all university music departments chords were regarded as originally produced by polyphonic movement of parts, yet during the oral part of degree testing at Durham that year, the external examiner (Head of Department at another university) asked a candidate to play the first chord in Mendelssohn's Wedding March, to name it and declare its root! (One of the home team was heard to say sotto voce: "I thought we'd done with all that dentistry!") There followed a long discussion of nomenclature, turning eventually upon the time-honored claims of this or that combination of notes to be called an added sixth. Apparently there were rival claimants for the rootship of the chord depending upon the name one chose to give it, and the candidate finally gave offense by admitting that he did not mentally name chords when he used them, and would teach them by figured bass if "the pupil were blockhead enough to need instruction before acquiring control of them". Fortunately for him, a dogfight among the examiners spent much of the time during which he might have recognised his own need for further instruction!

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LETTERS TO THE EDITOR

Sir:

Wilford W. Berard's article concerning the notation of the 11th and 13th partials (this journal, April, 1961) brought to my mind the fact that Bach recognized this ambiguity of pitch and used it to his advantage in his writing for the natural horn.

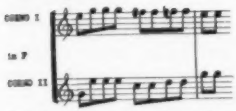
This instrument was, of course, limited to the production of the notes of the overtone series and in Bach's time the technique of "stopping" had not yet been adopted as a means of filling in this limited scale. In spite of this, Bach calls for chromatic and diatonic progressions of notes not found in the overtone series fairly often.

A look at the music will show that these progressions nearly always occur as inflections of notes produced as the 7th, 11th, or 13th partials of the series used as the acoustical basis of a natural horn. It seems that Bach recognized the fact that since these notes sound somewhere between two notes of acceptable pitch, they may be used with two different inflections: (1) that of the pitch with which they most closely correspond in equal temperament, and (2) that of the pitch toward which they are tending due to their deviation from the equal tempered scale.

For example, on the 11th partial the horn might be called upon to play a written F#. In another passage the same partial may be written as F♮. Since the 11th partial sounds somewhere between these tones it may be "lipped" up to sound F# and "lipped" down to sound F♮ without too much difficulty. (Written pitches are referred to in order to more readily show the relationship of these tones with Mr. Berard's series on Great C.)

The same principle may be used on the 13th partial. The note may be used as $A^{\flat}$, or since it is sharp, it may be inflected with the lips to produce an A♮.

Two short examples from Bach's Cantata No. 40 will illustrate this process.

Example 1

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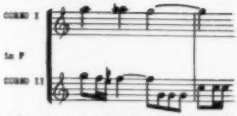
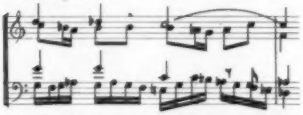
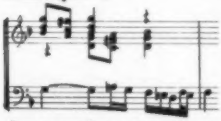


In these examples Bach does not hesitate to have the 1st horn change directly from one inflection of a partial to the other. Space does not permit further examples but the reader may find his own in the first Brandenburg Concerto and in the Quoniam of the B minor Mass. (In this last work the player must trill from G# (A^b to A.)

The E^b called for by the editor in the 2nd horn's part of the second example is another matter, however. Bach's own written $E\natural$ is the 10th partial and is not a false note. The change of this tone to E^b on the natural horn would be very difficult as it would amount to the production of a minor harmony from the overtone series. A check with the other instruments shows that the editor's emendation was apparently to avoid the simultaneous cross-relation resulting from the sounding A of the horn and the A^b of the bass.

Example 3

Bach does not mind this type of dissonance, however, and in a familiar example from the 2nd movement of the first Brandenburg Concerto he uses the same two notes in the same way. In both cases the dissonant tones are merely neighbor-tones ornamenting the chord tones G and B^b. The editor's sounding A^b is unnecessary.

Example 4

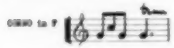


As may be seen in a comparison of the second and third examples, Bach has removed the horn from the part in question by the time the A^b has become a chord tone. This fact illustrates the observation that Bach expected his players to be able to inflect the 11th and 13th partials, but did not often

expect them to alter those partials that happened to be in tune.

The 7th partial is also frequently used in Bach's horn parts, but this note is not so often varied in pitch as partials 11 and 13. The 7th partial is flat by the standard of equal temperament and the logical inflection is a further flattening to produce a written A. This usage may be seen in the opening aria of the cantata, Was mir behagt. The trill from G to A is possible on the natural horn only by using a flattened form of the 7th partial as the note A.

Example 5



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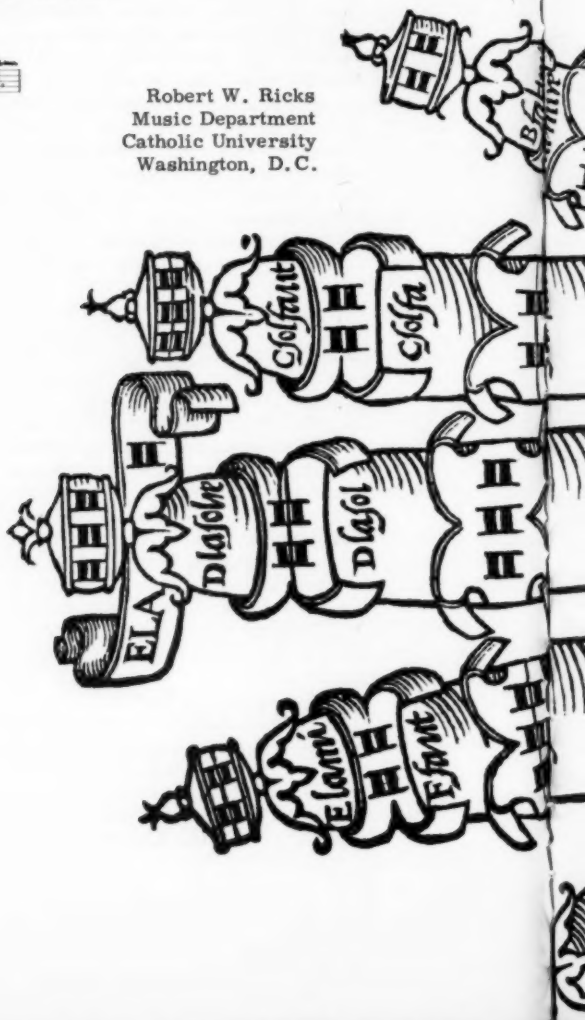

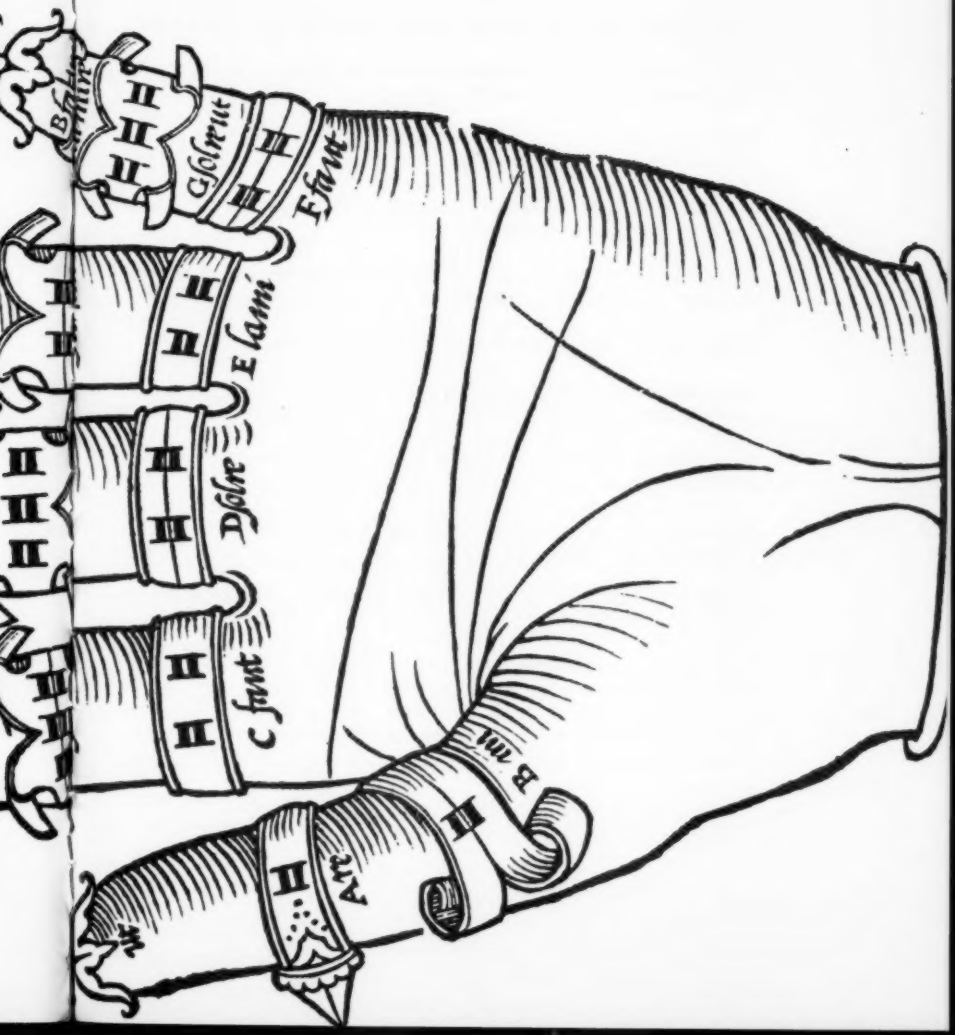
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